The paper was presented at the Tenth Meeting
"New Trends in Fatigue and Fracture" (NT2F10)
Metz, France, 30 August–1 September, 2010

Yu. G. Matvienko

DEVELOPMENT OF MODELS AND CRITERIA OF NOTCH FRACTURE MECHANICS

RAZVOJ MODELA I KRITERIJUMA MEHANIKE LOMA ZAREZA

Original scientific paper
UDC: 539.42.013.3
Paper received: 31.01.2011

Author's address:
Mechanical Engineering Research Institute of the Russian Academy of Sciences, 4 M. Kharitonievsky Per., 101990 Moscow, Russia, e-mail: matvienko7@yahoo.com

Keywords
- notch failure assessment diagram
- notch fracture toughness
- constraint factor
- 3D effective distance
- J-integral

Abstract

Models and criteria of fracture mechanics in terms of the notch stress intensity factor and J-integral are adopted for describing deformation and fracture of a solid with a notch.

The notch failure assessment diagram is constructed as the unified diagram for a notch as well as a crack. Effects of local constraint are incorporated into basic criteria equations which allow estimating the notch constraint-dependent fracture toughness and notch failure assessment diagrams for various components with a notch and various types of loading.

The volumetric method suggested has been discussed from a viewpoint of 3D finite element analysis of the elastic-plastic stress along the crack front under uniaxial and biaxial mode I loading.

Calculation of the J-integral in plates with lateral and central U- and V-blunt notches under mode I loading can be based on approximate analytical formulas taking into account both a linear elastic and nonlinear elastic material.

The load separation concept has been described to measure the notch fracture toughness using non-standard notched specimens and the mixed (modes I+II) J-integral in the case of the tension plate with an inclined centre crack for power law hardening material.

Ključne reči
- dijagram ocene otkaza sa zarezom
- žilavost loma sa zarezom
- faktor veze
- 3D efektivno rastojanje
- J integral

Izvod

Modeli i kriterijumi mehanike loma u kojima se koriste faktor intenziteta napona sa zarezom i J integral su usvojeni za opisivanje deformacije i loma čvrstog tela sa zarezom.

Dijagram ocene otkaza sa zarezom konstruisan je kao unificirani dijagram kako za zarez tako i za prslinu. Uticaji lokalne veze uneti su u jednačine osnovnih kriterijuma kojima se određuje zarezna žilavost loma, zavisna od veza, kao i dijagrame ocene otkaza sa zarezom i raznim tipovima opterećenja.

Predloženi volumetrijski metod je razmatran sa gledišta analize 3D konačnim elementima elastoplastičnog napona duž fronta prsline sa jednoosnim i dvoosnim opterećenjem u modu I.

Proračun J integrala za ploče sa bočnim i centralnim U i V zarezima pod dejstvom opterećenja u modu I se može bazirati na približnim analitičkim izrazima koji uzimaju u obzir linearno elastičan i nelinearno elastičan materijal.

Opisana je koncepcija razdvajanja opterećenja radi merenja zarezne žilavosti loma upotrebom nestandardnih zarezanih epruveta kao i mešovitog (u modu I+II) J integrala u slučaju zatezne ploče sa nagnutom središnjom prslinom kod materijala koji ojačava po eksponencijalnom zakonu.

INTRODUCTION

At the present time, fracture mechanics principles are applied to study stress distribution in the vicinity of the notch tip and for describing failure of components with notch-like defects. For example, the basic failure assessment diagram has been modified using the concept of the notch stress intensity factor. In this case, the so-called notch failure assessment diagram (NFAD) is written in terms of K_{notch}/K_{Nmat} and σ_C/σ_Y for a notch-like defect taking into account a finite notch tip radius ρ, /1-5/, and can be used for structural integrity assessment of a component. Here, K_{Nmat} is the notch fracture toughness, σ_C is the failure stress, σ_Y is the yield strength. The stress intensity factor K at the notch tip is denoted as K_{notch}. The notch fracture toughness, which is applied to the NFAD, should be calculated or measured for a structural component. Moreover, the FITNET assessment of a structural component by the standard and advanced J-integral based options also requires notch fracture toughness data in terms of the J-integral derived from tests of notched specimens.

The aim of this paper is to give a brief survey of some basic principles of notch fracture mechanics to describe the notch failure assessment diagram and to measure the notch fracture toughness for a component with a notch-like defect. In the present paper the notch failure assessment diagram suggested by Matvienko, /1-4/, has been employed for discussion.

NOTCH FAILURE ASSESSMENT DIAGRAM

Basic failure criteria

The local failure criterion, averaging the normal elastic stress $\sigma_y(r)$ ahead of the notch tip over the fracture process zone length d, can be given as follows

$$\frac{1}{d}\int_0^d \sigma_y(r)dr = \sigma_0 \tag{1}$$

and allows to describe a critical state of a solid with a notch. Here, σ_0 is the local strength (called the cohesive strength in previous papers, /1-3/) within the fracture process zone.

The criterion of the average stress, Eq.(1), has been adopted for a body with a notch, as well as a crack. In this case, the following assumption has been made. The normal stress distribution at the notch tip is similar to that crack under uniform remote tensile stress but shifted from the notch tip to a point of abscissa to $\rho/2$, i.e. $r \geq \rho/2$. It should be pointed out that such stress distribution is suggested for blunt cracks when the distance ahead of the crack tip is much smaller than the crack length and greater than the crack tip radius. The stress distribution is simplified considerably on the continuation of the U-notch, /6/,

$$\sigma_y(r) = \frac{K_{notch}}{\sqrt{2\pi r}}\left(1+\frac{\rho}{2r}\right). \tag{2}$$

Averaging the local stress, Eq.(2), over the fracture process zone, the fracture criterion Eq.(1) leads to the notch failure assessment diagram in terms of the stress intensity factor and applied stresses σ_C, /1-4/,

$$K_{notch} = K_{mat}\sqrt{1-\left(\frac{\sigma_C}{\sigma_0}\right)^2}\left[1-\left(\frac{\sigma_0}{\sigma_C}\right)^2\frac{1}{K_t^2}\right]^{-1/2}. \tag{3}$$

Here, K_{mat} is the fracture toughness, K_t is the elastic stress concentration factor.

The local strength within the fracture process zone

The local strength σ_0 is treated according to von Mises yield criterion as a property of both the yield stress and the T-stress which was introduced into the criterion to quantify constraint in different geometries and type of loading. The local strength for finite geometries was rewritten as a function of the applied failure stress and the notch (or crack) tip constraint characterized by a dimensionless parameter β (so-called biaxiality ratio) which depends on geometry and loading mode /3, 4/. Finally, the local strength σ_0 can be expressed as follows

$$\frac{\sigma_0}{\sigma_Y} = -\frac{\beta}{2}\left(\frac{\sigma_C}{\sigma_Y}\right)+\sqrt{\frac{1}{4}\left(\frac{\beta\sigma_C}{\sigma_Y}\right)^2-\frac{(1+\nu^2-\nu)\,(\beta\sigma_C/\sigma_Y)^2-1}{(1-2\nu)^2}} \tag{4}$$

for the case of plane strain and

$$\frac{\sigma_0}{\sigma_Y} = -\frac{\beta}{2}\left(\frac{\sigma_C}{\sigma_Y}\right)+\sqrt{1-\frac{3}{4}\left(\frac{\beta\sigma_C}{\sigma_Y}\right)^2} \tag{5}$$

for the case of plane stress.

For the classical infinite plate under uniaxial loading $\beta = -1$, and special case of the local strength occurs.

The notch fracture toughness and the NFAD

Basic failure criterion Eq.(3) for describing the NFAD can be also employed to make an interpretation of the notch fracture toughness as follows

$$K_{Nmat} = K_{mat}\left[1-\left(\frac{\sigma_0}{\sigma_C}\right)^2\frac{1}{K_t^2}\right]^{-1/2}. \tag{6}$$

Thus, the notch fracture toughness can be written as a function of the fracture toughness K_{mat}, the elastic stress concentration factor K_t, the local strength σ_0 and the applied failure stress σ_C. Equation (6), describing the notch fracture toughness, suggests that the loss of constraint due to a notch (K_t) is independent on the loss of constraint due to the T-stress (or biaxiality parameter β) which was introduced into the local strength σ_0 (Eqs. (4) and (5)) to quantify constraint in different geometries and type of loading. In this case, the value of K_{Nmat} could be considered as the notch constraint-dependent fracture toughness K^C_{Nmat} which is transferred in the constraint-dependent fracture toughness K^C_{mat} for a crack.

It should be noted that the SINTAP procedure has been modified using the concept of the notch stress intensity factor and a NFAD, /5/. Some expressions for notch fracture toughness have been proposed in Refs. /7, 8/.

Taking into account Eqs. (3) and (6), the NFAD should be represented by the following equation

$$K_{notch} = K_{Nmat}\sqrt{1-\left(\frac{\sigma_C}{\sigma_0}\right)^2}. \tag{7}$$

Consideration of a crack as a special case of a notch ($K_t \to \infty$) changes the notch fracture toughness K_{Nmat} (Eq. (6)) into the fracture toughness K_{mat} (for a crack) and Eq. (7) can be rewritten for the crack

$$K = K_{mat}\sqrt{1-\left(\frac{\sigma_C}{\sigma_0}\right)^2}. \tag{8}$$

It means that the proposed NFAD is the unified failure assessment diagram for a notch as well as a crack. Difference of these two cases is just connected with calculation of fracture toughness (K_{mat} or K_{Nmat}) and a position of the corresponding points for the notch and crack of same length on the failure assessment curve.

In contrast to the SINTAP procedure and FITNET assessment, this simple methodology does not require finite element analysis to obtain the effective distance for calculating the notch failure assessment diagram.

Validation of the present FAD

It should be noted that reports in literature of full-scale experiments with information on the initiation or failure load (or stress) for specimens with cracks and notches under different types of loading are limited.

The validation study is made on through-cracked plates made of different materials at different temperatures. The fracture toughness K_{mat} is evaluated from Eq. (8) employing the fracture data and calculated local strength. The failure assessment diagram is constructed using these parameters. The experimental constraint-dependent fracture toughness K^C_{mat} is calculated for these cracked plates subjected to a uniform failure tensile stress σ_C using the well-known equation for the stress intensity factor K.

A comparison between the variation of the predicted constraint-dependent fracture toughness K^C_{mat} with failure stress, the results of SINTAP procedure and experimental results shows good agreement for $0.8 \leq \sigma_C/\sigma_Y$ (Fig. 1), /3/.

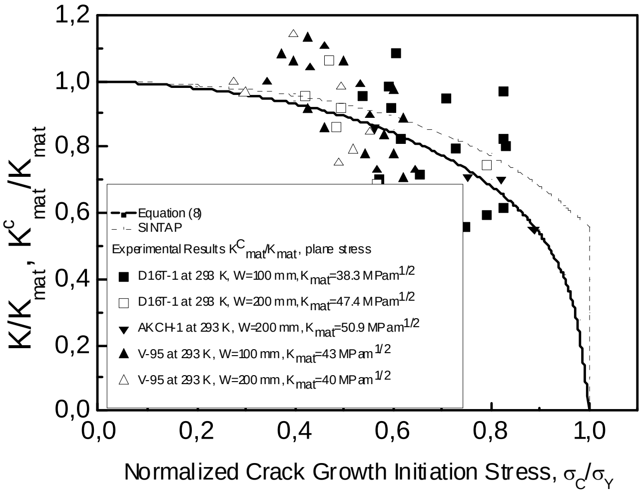

Figure 1. Comparison of predicted and experimental results of the failure assessment diagram at crack growth initiation for aluminium alloy centre-cracked tensile specimens at 293 K.
Slika 1. Poređenje rezultata procene i eksperimenata dijagrama ocene otkaza za inicijaciju rasta prsline u zategnutim epruvetama legure aluminijuma sa središnjom prslinom, na 293 K.

THE VOLUMETRIC METHOD

Background of the method

The concept of the volumetric method, suggested by Pluvinage /9/, assumes that the fracture process requires a certain fracture process volume. This volume is assumed as a cylinder with a diameter called the effective distance. The notch stress intensity factor is described by the effective stress σ_{eff} and the effective distance X_{eff} and can be used for measurement of the notch fracture toughness. According to this approach, determination of the effective distance is based on the bi-logarithmic stress distribution along the ligament ahead of the notch tip and requires finite element analysis. Linear behaviour in the bi-logarithmic diagram is observed beyond the highest stressed zone and starts at a certain distance, which is called the effective distance. The effective distance corresponds to the inflexion point with the minimum of the relative opening stress gradient. The notch stress intensity factor K_{notch} describes the stress distribution outside the zone characterized by X_{eff} and can be defined as a function of the effective distance and effective stress in the case of the U-notch (with a notch angle $\varphi = 0$) as follows

$$K_{notch} = \sigma_{eff}\sqrt{2\pi X_{eff}} \,. \tag{9}$$

Here, the effective stress σ_{eff} is considered as the average value of the stress distribution within the fracture process zone. The notch stress intensity factor K_{notch} reaches the critical value, i.e. the notch fracture toughness $K_{Nmat} = \sigma_{eff,c}\sqrt{2\pi X_{eff,c}}$, when failure occurs.

3D analysis of effective distance

Until now the effective distance has been estimated by means of 2D finite element analysis. Recently, full-field 3D finite element analysis is carried out to determine the elastic-plastic and creep stress along the through-thickness inclined crack front in a circular disk subjected to biaxial mixed mode loading /10/. The strain hardening exponent n for the material is assumed to be 4.96. Some results of calculations for biaxial tension-compression ($\eta = -1$), uniaxial tension ($\eta = 0$) and biaxial tension-tension ($\eta = +1$) under pure mode I loading are given below.

It has been observed that the effective distance is related to the minimum value of the relative stress gradient under all considered biaxial loading conditions. The effective distance distribution is plotted as a function of a position of points on the crack front characterized by dimensionless coordinate (z/b), where b is thickness (Fig. 2), /10/. It can be seen that the effective distance increases dramatically along the crack front from free surface ($z/b = 0$) to the mid-plane of plate thickness ($z/b = 0.5$). Stabilization of the effective distance distribution is observed at the value of $z/b \approx 0.15$–0.25. Moreover, there is the effect of biaxiality on the effective distance and its stabilization.

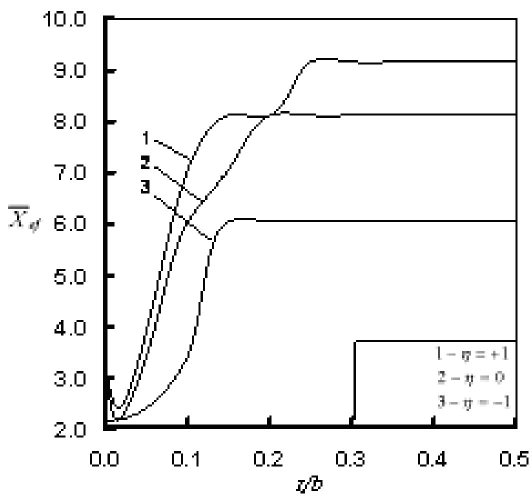

Figure 2. Distribution of the effective distance along the through-thickness crack front.
Slika 2. Raspodela efektivnog rastojanja duž fronta prolazne prsline.

Similar behaviour of the effective distance can be expected to be along the notch front. Thus, 3D analysis of the effective distance distribution along the notch front could be very attractive to realize an idea of the volumetric approach suggested by Pluvinage /9/, and to estimate the notch fracture toughness.

J-INTEGRAL

The expression of the J-integral as a function of strain energy over U- and V-notch edge, considering only the notch arc contribution of a blunt V-notch (and excluding the contribution of rectilinear flanks), can be written in the case of a generic opening angle 2α (different from zero) as follows

$$J=\int_{-\pi/2+\alpha}^{\pi/2-\alpha} W(\theta)\,\rho\cos(\theta)d\theta\,. \tag{10}$$

The integration path is assumed to be coincident with the semi-circular arc of the notch, which is traction free. For a numerical investigation of strain energy density distribution on the notch edge, the equation

$$W(\theta)=W_{\max}\cos^{\delta}(\theta) \tag{11}$$

is assumed, where δ is determined from finite element analyses /11/. A multi-parametric analysis in the case of a plate weakened by lateral and central U-blunt notches under mode I loading is carried out considering a large variability of the notch acuity $4 \leq a/\rho \leq 400$ and the opening angle $0 \leq 2\alpha \leq 3\pi/4$ taking into account both a linear elastic and non-linear elastic material, the latter being modelled according to a power hardening law. In order to analyse the effect of different load intensities on strain energy density and J-integral formulations, different stress levels σ/σ_Y are applied to the plates. The results from finite element analysis demonstrated that the exponent δ depends on the notch opening angle and notch acuity a/ρ. It does not depend on the stress level and the strain hardening exponent. The basic equations for exponent δ are summarized in tabular form /11/. The predicted results of the J-integral are consistent with those directly obtained from finite element analyses.

LOAD SEPARATION METHOD

The notch fracture toughness can be estimated in terms of the J-integral. To measure notch fracture toughness $J_{\rho,c}$ and calculation of J-integral updated for crack growth in the case of different materials, the test procedure based on the load separation method is employed to determine the η_{pl}- and η-factor of the non-standard curved notched specimen, namely, the CT specimen and arched specimen (so called "Roman tile") under three-point bending. In this case, the original representation of the J-integral as total energy release rate and tests records, namely, load versus total load-line displacement, were used taking into account an equality between the η_{pl}- and η-factors. It is shown that the proposed procedure allows avoiding calculation of the stress intensity factor for non-standard specimens to determine the notch fracture toughness $J_{\rho,c}$. The η-factor (η_{pl}-factor) should be estimated by testing at least 3 specimens with different notch aspect ratio. Moreover, the load separation method allows predicting the growing crack length and constructing the J-R curve for these non-standard specimens. Details of the experimental procedures and results are given in Refs. /12, 13/.

The load separation concept and finite element method are also used to determine mixed (modes I + II) mode plastic η_{pl} and η_{pl}^{COD} factors for the tension plate with an inclined centre through-thickness crack in the case of power law hardening materials, /14/. The calculated plastic η_{pl} and η_{pl}^{COD} factors decrease with increase of crack orientation angle θ (from pure mode I ($\theta = 0$) to mixed (modes I + II) mode) and the decrease of the strain hardening exponent (Fig. 3). It is shown that normalized representation of the mixed mode plastic factor $\eta_{pl}(\theta)/\eta_{pl}(\theta = 0)$ allows avoiding the dependence of these values on specimen configuration. The obtained η_{pl} and η_{pl}^{COD} factors for mixed loading should be very useful to validate the J-integral criteria of a mixed crack initiation as well as to measure the mixed fracture toughness.

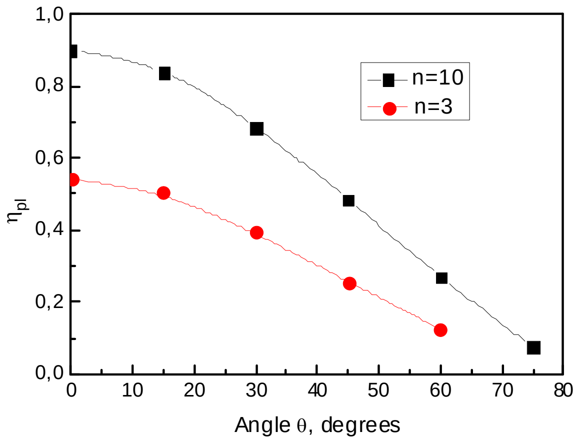

Figure 3. Mixed mode plastic factors for the tension plate with an inclined centre crack for power law hardening material.
Slika 3. Faktori plastičnosti za mešoviti tip loma zatezne ploče sa nagnutom središnjom prslinom kod materijala koji ojačava eksponencijalno.

CONCLUSION

A brief survey of some basic principles of notch fracture mechanics is discussed for describing the notch failure assessment diagram and to measure the notch fracture toughness for a component with a notch-like defect. The notch stress intensity factor and the J-integral are employed as basic parameters of notch fracture mechanics.

It is shown that the proposed notch failure assessment diagram can be constructed as the unified diagram for a notch as well as a crack. The basic failure criterion, describing the notch fracture toughness, suggests that the loss of constraint due to a notch is independent on the loss of constraint due to the T-stress (or biaxiality parameter β) which is introduced into the local strength σ_0 to quantify constraint in different geometries and type of loading. This statement allows considering the notch fracture toughness as the notch constraint-dependent fracture toughness K^c_{Nmat}. The validation study of the proposed failure assessment diagram is made on through-cracked plates made of different materials at different temperatures.

The volumetric method suggested by Pluvinage (9) is discussed from a viewpoint of 3D finite element analysis of elastic-plastic stress along the crack front under uniaxial and biaxial mode I loading. The distribution of the effective distance along the crack front is observed. Similar behaviour of the effective distance can be expected to be along the notch front. Thus, 3D analysis of the effective distance distribution along the notch front could be very attractive to realise an idea of the volumetric approach and to estimate the notch fracture toughness.

Some approximate formulas to calculate the J-integral in plates with lateral and central U- and V-blunt notches under mode I loading have been obtained taking into account both a linear elastic and nonlinear elastic material.

Experimental procedure based on load separation method is developed that allows avoiding calculation of the stress intensity factor for non-standard notched specimens to measure the notch fracture toughness and to estimate the J-integral in case of mixed (modes I + II) mode loading.

ACKNOWLEDGEMENT

This research is carried out with the support of Russian Foundation of Basic Research (Grant No 10-08-00393).

REFERENCES

1. Matvienko, Yu.G., *Local fracture criterion to describe failure assessment diagrams for a body with a crack/notch*, Int J Fract, 2003, pp.107-112.
2. Matvienko, Yu.G., *Erratum: Local fracture criterion to describe failure assessment diagrams for a body with a crack/notch*, Int J Fract, 2005, p.309.
3. Matvienko, Yu.G., *On Constraint-modified failure assessment diagrams for solids with a crack/notch*, Proc. of 17th ECF "Multilevel Approach to Fracture of Materials, Components and Structures", 2-5 Sept. 2008, Brno, Czech Republic.
4. Matvienko, Yu.G., *Failure assessment diagrams in structural integrity analysis*, "Damage and Fracture Mechanics - Failure Analysis of Engineering Materials and Structures", Eds. T. Boukharouba, G. Pluvinage, M. Elboujdaini, Springer, 2009, pp.173-182.
5. Adib, H., Jallouf, S., Schmitt, C., Carmasol, A., Pluvinage, G., *Evaluation of the effect of corrosion defects on the structural integrity of X52 gas pipelines using the SINTAP procedure and notch theory*, Int J Pressure Vessels Piping, 2007, pp.123–131.
6. Creager, M., Paris, P.C., *Elastic field equations for blunt cracks with reference to stress corrosion cracking*, Int J Fract, 1967, pp.247-252.
7. Taylor, D., Cornetti, P., Pugno, N., *The fracture mechanics of finite crack extension*, Eng Fract Mech, 2005, pp.1021-1038.
8. Kim, J.H., Kim, D.H., Moon, S.I., *Evaluation of static and dynamic fracture toughness using apparent fracture toughness of notched specimen*, Mater Sci Eng A, 2004, pp.381-384.
9. Pluvinage, G., Fracture and Fatigue Emanating from Stress Concentrators, Dordrecht, Kluwer Academic Publishers, 2003.
10. Shlyannikov, V.N., Matvienko, Yu.G., Boichenko, N.V., Tartygasheva, A.M., *The effective distance and T-stress in cracked plastic and creeping solids under biaxial loading*, Proc. of 18th ECF "Fracture of Materials and Structures from Micro to Macro Scale", 30 Aug.-3 Sept. 2010, Dresden, Germany.
11. Berto, F., Lazzarin, P., Matvienko, Yu.G., *J-integral evaluation for U- and V-blunt notches under Mode I loading and materials obeying a power hardening law*, Int J Fract, 2007, pp.33-51.
12. Kim, Y.S., Matvienko, Yu.G., Jeong, H.C., *Development of Experimental Procedure Based on the Load Separation Method to Measure the Fracture Toughness of Zr-2.5Nb Tubes*, Key Engineering Materials, 2007, pp.449-452.
13. Capelle, J., Gilgert, J., Matvienko, Yu.G., Pluvinage, G., Measurement of the Resistance to Fracture Emanating from Scratches in Gas Pipes Using Non-standard Curved Specimens, "Security and Reliability of Damaged Structures and Defective Materials", Eds. G. Pluvinage and A. Sedmak, Springer Netherlands, 2009, pp.157-174.
14. Matvienko, Yu.G., Muravin, E.L., *Load separation method in plastic J-integral estimation for inclined cracks*, Proceeding of International Conference on Crack Paths (CP 2009), 23-25 Sep. 2009, Vicenza, Italy, pp.455-462.