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# UTICAJ SOPSTVENIH UČESTANOSTI NA INICIJACIJU PRSLINA KUĆIŠTA ZUPČASTE PUMPE

# INFLUENCE OF EIGEN FREQUENCIES ON CRACK INITIATION IN GEAR PUMP HOUSING

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**Ključne reči**
- inicijacija prslina
- kućište zupčaste pumpe
- dinamička analiza
- sopstvena učestanost
- sopstveni modovi oscilovanja

*Izvod*

*Razmotreni su osnovni teorijski aspekti dinamičke analize konstrukcija i analiziran poseban primer kućišta jedne zupčaste pumpe visokog pritiska. Model konstrukcije je baziran na 3D elementima.*

*Korišćen je programski paket COSMOS. Dobijeni rezultati, na nivou osnovnog dinamičkog ponašanja konstrukcije kućišta sa realnim konturnim uslovima, odnose se na sopstvene učestanosti i sopstvene modove oscilovanja, koji su prikazani u grafičkoj formi.*

**Keywords**
- crack initiation
- gear pump housing
- dynamic analysis
- eigen frequency
- eigen modes of oscillation

*Abstract*

*The theoretical aspects of dynamic structural analysis are considered and a numerical example is analyzed for the high pressure gear pump housing. The structural model is based on 3D elements.*

*The software package COSMOS is applied. The obtained results, based on the fundamental level of the housing structural dynamic behaviour with real boundary condition, refer to eigen frequencies and eigen modes of oscillations, that are represented in graphic forms.*

## UVOD

Radi uvida u potencijalno kritična mesta od interesa za određivanje zona visokog nivoa složenih dinamičkih napona i moguće inicijacije prslina na kućištu zupčaste pumpe, analiziran je 3D model za identifikaciju sopstvenih učestanosti i sopstvenih modova oscilovanja.

Zupčaste pumpe su široko primenjene u mašinskim konstrukcijama. Zbog specifičnosti takvih konstrukcija u pogledu radnih uslova, uređaji moraju biti precizno definisani, proračunati, izvedeni i ispitani, kako bi se mogao garantovati visok nivo pouzdanosti performansi u eksploataciji.

Kada su u pitanju konstrukcijske komponente pumpe, kao što je na primer kućište, posebna pažnja mora se posvetiti izboru njegove geometrije, materijala i raspodele mase radi zadovoljenja kriterijuma otpornosti (dovoljna krutost, projektovano dinamičko ponašanje, dopušteni naponi), uštede u materijalu (smanjena masa), smanjenje nivoa buke (uvek prisutan zahtev za hidrauličke i pneumatske uređaje). Naravno, mora se voditi računa i o ceni proizvoda, pa se zato performanse uređaja, ipak, postižu kao kompromis na bazi geometrijskih, materijalnih i ekonomskih kriterijuma. U skladu sa napred navedenim, ustanovljen je i NIR program I.5.1781, u okviru koga su ostvareni zapaženi rezultati u optimizaciji i analize konstrukcije pumpe primenom metode konačnih elemenata, kao i rezultati u pogledui prostornog ispitivanja buke. Definisan je i predlog mera za smanjenje mase i nivoa buke.

## INTRODUCTION

An overview of potential critical locations of interest for determining the zones of high level complex dynamic stresses, that may lead to crack initiation in gear pump housing, has included in the analysis of a 3D model for identifying eigenfrequencies and eigen modes of oscillation.

Gear pumps are widely applied in mechanical structures. Characteristics of these structures with respect to operating conditions must rely on devices that are accurately defined, calculated, constructed and tested, as to assure the high level of reliability of performances in service.

When pump structural components are in question, as in the case of pump housing, special attention must be paid to optimal geometry, material and mass distribution, all in the goal of fulfilling the resistance criteria (sufficient stiffness, designed dynamic behaviour, allowed stresses), savings in material (mass reduction), noise reduction (always present requirement for hydraulic and pneumatic devices). Of course, product cost is also an issue, and so device performance is reached yet as a compromise based on the criteria concerning geometry, materials and economy. In accordance with the above mentioned, a NIR programme I.5. 1781 is established, with distinctive results concerning optimisation and pump structural analysis, by applying the finite element method, as well as results concerning spatial noise investigation. Some measures for reducing the mass and noise level are also defined and proposed.

U ovom radu, biće više reči o delimičnoj dinamičkoj analizi konstrukcije, gde se kao cilj postavlja definisanje pogodnog proračunskog modela, koji omogućuje identifikaciju odziva konstrukcije u celini, a ne samo nalaženje ekstremnih vrednosti za neke radne uslove, kao što je slučaj kod idealizovanih modela pri konvencionalnom proračunu.

Ovde će se postaviti analiza osnovnih dinamičkih parametara realne konstrukcije opšte materijalne i geometrijske konfiguracije. Najpre će se postaviti relacije za identifikaciju dinamičkog ponašanja konstrukcijskg sistema, a zatim relacije za određivanje sopstvenih učestanosti i sopstvenih modova oscilovanja konstrukcije.

Analiza će biti bazirana na konačnim 3D elementima sa 12 stepeni slobode /2, 3/.

## OSNOVNE TEORIJSKE JEDNAČINE

Na bazi teorijskih razmatranja, saglasno /1, 2, 3/, kao osnovni model iskoristiće se statički model sa odgovarajućim brojem i vrstom konačnih elemenata i formiranim matricama krutosti, inercije i prigušenja konstrukcije, na nivou elemenata i konstrukcije u celini.

U uslovima kada je vektor čvornih sila vremenska funkcija, $\{F(t)\}$, dinamička analiza konačnog elementa izvodi se na osnovu jedn. (1), koja je rezultat jednakosti radova spoljnjih sila pri virtualnim pomeranjima, /3/.

$$[m]_{(k)}\frac{\partial^2}{\partial t^2}\{\delta\}_{(k)}+[b]_{(k)}\frac{\partial}{\partial t}\{\delta\}_{(k)}+[k]_{(k)}\{\delta\}_{(k)}=\{F_{(t)}\}_{(k)} \quad (1)$$

gde su: $[m]_{(k)}$ – matrica inercije elemenata; $[b]_{(k)}$ – matrica prigušenja konstrukcijskih elemenata; $[k]_{(k)}$ – matrica krutosti elemenata; $\{\delta\}_{(k)}$ – pomeranje elemenata; $\partial/\partial t$, $\partial^2/\partial t^2$ – parcijalni izvodi; $\{F(t)\}_{(k)}$ – matrica opterećenja.

Shema sastavljanja matrica inercije– [M], krutosti–[K], prigušenja konstrukcije–[B], pomeranja–[δ] i opterećenja–[F] na nivou konstrukcije u celini je poznata /3, 4/.

Nakon formiranja navedenih matrica, može se definitivno zapisati jednačina prinudnih oscilacija noseće konstrukcije, diskretizovane konačnim elementima,

$$[M]\frac{\partial^2}{\partial t^2}\{\delta\}+[B]\frac{\partial}{\partial t}\{\delta\}+[K]\{\delta\}=\{F\} \quad (2)$$

Kako je u opštem slučaju osnovna jednačina problema određivanja sopstvenih parametara oscilovanja sistema konstrukcije /3/,

$$[M]\frac{\partial^2}{\partial t^2}\{\delta\}+[K]\{\delta\}=\{0\} \quad (3)$$

a izrazi za karakteristična pomeranja čvorova,

$$\{\delta_i(t)\}=\{\delta_i^0\}e^{i\omega t} \quad (4)$$

gde je $\omega$–sopstvena kružna učestanost, a $t$–vreme, tada se može pisati,

$$\left(-\omega^2[M]+[K]\right)\{\delta_i^0\}=0 \quad (5)$$

pa se rešenja za jedn. (5) mogu tražiti iz uslova da je vrednost determinante D = 0,

$$D=\det\left|-\omega^2[M]+[K]\right|=0 \quad (6)$$

This paper relates more to partial dynamic structural analysis, where defining a competing calculation model, that is capable of identifying the response of the whole structure and not only determining extreme values for some operating conditions, as is the case for idealized models in conventional calculations.

An analysis of basic dynamic parameters of a real structure with a general configurations of material and geometry are assumed here. Relations for identifying dynamic behaviour of the structural system are postulated first, followed by relations for determining eigen frequencies and eigen modes of oscillation.

Analysis is based on finite element method using 3D elements with 12 degrees of freedom /2, 3/.

## BASIC THEORETICAL EQUATIONS

Based on theoretical considerations, in accordance with /1, 2, 3/, as a basic model, the static model is used with an appropriate number and type of finite elements and already formed matrices of stiffness, inertia and structural attenuation, at the level of elements and the whole structure.

In conditions when the vector of nodal forces is a time function, $\{F(t)\}$, the dynamic analysis of the finite element is performed based on Eq. (1), resulting from the equality of work by external forces in virtual displacement, /3/.

$$[m]_{(k)}\frac{\partial^2}{\partial t^2}\{\delta\}_{(k)}+[b]_{(k)}\frac{\partial}{\partial t}\{\delta\}_{(k)}+[k]_{(k)}\{\delta\}_{(k)}=\{F_{(t)}\}_{(k)} \quad (1)$$

where: $[m]_{(k)}$ – element inertia matrix; $[b]_{(k)}$ – matrix of the structural attenuation of elements; $[k]_{(k)}$ – element stiffness matrix; $\{\delta\}_{(k)}$ – element displacement; $\partial/\partial t$, $\partial^2/\partial t^2$ – partial differentials; $\{F(t)\}_{(k)}$ – load matrix.

The scheme for composing matrices of inertia– [M], stiffness–[K], structural attenuation–[B], displacement–[δ] and load–[F] at the level of the whole structure is known /3, 4/.

After these matrices are formed, the equation of forced oscillations of the loaded structure may be written in discretized finite elements,

$$[M]\frac{\partial^2}{\partial t^2}\{\delta\}+[B]\frac{\partial}{\partial t}\{\delta\}+[K]\{\delta\}=\{F\} \quad (2)$$

Since in the general case, the basic equation for problems in determining eigen parameters of oscillation in the structured system is, /3/,

$$[M]\frac{\partial^2}{\partial t^2}\{\delta\}+[K]\{\delta\}=\{0\} \quad (3)$$

and relations for characteristic node displacement are

$$\{\delta_i(t)\}=\{\delta_i^0\}e^{i\omega t} \quad (4)$$

where: $\omega$–eigen angular frequency, and $t$–time, then it may be written,

$$\left(-\omega^2[M]+[K]\right)\{\delta_i^0\}=0 \quad (5)$$

and solutions for Eq. (5) are found from the condition of the determinant value D = 0,

$$D=\det\left|-\omega^2[M]+[K]\right|=0 \quad (6)$$

Zavisnosti između rešenja jedn. (6) su poznate iz teorije oscilacija /3, 4/.

Zamenom izračunatih vrednosti pojedinih sopstvenih učestanosti $\omega_i$ u izraz (5), uz odgovarajuće normiranje sistema homogenih algebarskih jednačina, npr. pomeranjem $\delta_1 = 1$, dobija se modalna matrica [μ], na osnovu koje je moguće grafički prikazati sopstvene modove oscilovanja.

$$[\mu]=\begin{bmatrix} 1 & 1 & \ldots & 1 \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ . & . & \ldots & . \\ \mu_{n1} & \mu_{n2} & \cdots & \mu_{nn} \end{bmatrix} \quad (7)$$

Svaki mod oscilovanja može se napisati kao

$$\{\mu_i\}=\{1 \quad \mu_{2i} \quad \mu_{3i} \quad .......\mu_{ni}\}^T \quad (8)$$

PRIMER

Model kućišta pumpe prikazan je na sl. 1. Primenjeni konačni elementi kojima je diskretizovana konstrukcija prikazani su na sl. 2. Razmotrene su tri različite konfiguracije kućišta, kako bi se minimizirali mase i troškova proizvodnje /5/.

Rezultati, u vidu nekoliko modova oscilovanja sa odgovarajućim kružnim frekvencijama, prikazani su na sl. 3 /6/.

Relations that connect solutions of Eq. (6) are known from the Theory of oscillations, /3, 4/.

By assigning calculated values to some eigen frequencies $\omega_i$ in Eq. (5), and when the system of homogeneous algebraic equations is normalized, e.g. with displacement $\delta_1 = 1$, the modal matrix [μ] is obtained, that allows a graphic representation of eigen modes of oscillation, as

$$[\mu]=\begin{bmatrix} 1 & 1 & \ldots & 1 \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ . & . & \ldots & . \\ \mu_{n1} & \mu_{n2} & \cdots & \mu_{nn} \end{bmatrix} \quad (7)$$

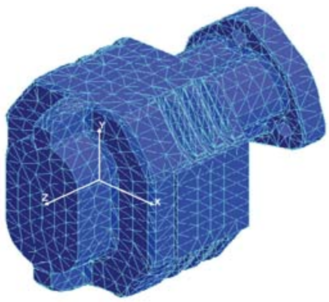
Every oscillating mode may be written as

$$\{\mu_i\}=\{1 \quad \mu_{2i} \quad \mu_{3i} \quad .......\mu_{ni}\}^T \quad (8)$$

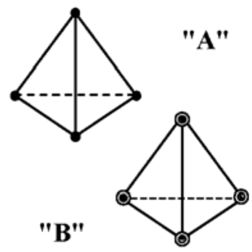
EXAMPLE

The gear pump housing model is shown in Fig. 1. Applied finite elements for structural discretization are shown in Fig. 2, /5/. Three different housing configurations are considered, for sake of minimizing the mass and manufacturing costs /5/.

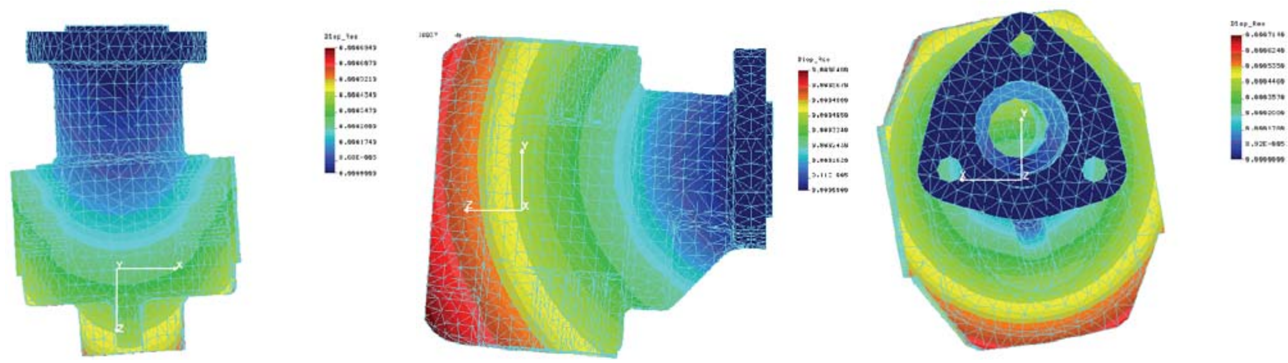
The results, represented in several oscillating modes with corresponding angular frequencies, are shown in Fig. 3 /6/.



Slika 1. Strukturalni model kućišta
Figure 1. Structural model of the housing.




Slika 2. Konačni elementi
Figure 2. Finite elements.



I–flex. $\omega_{1f,zx} = 458$ Hz II–flex. $\omega_{2f,zy} = 520$ Hz I–tors. $\omega_{1t,z} = 714$ Hz

Slika 3. Modovi oscilovanja za konfiguraciju „B“.
Figure 3. Configuration “B” modes of oscillation.

Izvedena analiza konstrukcije 3D modela je jednoznačno pokazala da se na racionalan način mogu pouzdano utvrditi sopstvene vrednosti (sopstvene učestanosti i sopstveni modovi oscilovanja), kao i zone visokog nivoa složenog naprezanja konstrukcije (kritične zone) u kojima se može očekivati iniciranje prsline kod kućišta zupčaste pumpe.

Performed structural 3D model analysis has uniquely displayed that eigen values (eigen frequencies and eigen modes of oscillation) can be identified in a rational way to a very reliable extent, as well as high level complex stress zones in a structure (critical zones), where crack initiation in a gear pump housing is most likely to occur.

Predmetna dinamička analiza konstrukcije kućišta (kao reprezentativnog modela hidrauličkih uređaja) pokazuje pogodnost primene pri rešavanju problema koji se mogu u različitim konstrukcijama hidrauličkih sistema i instalacija.

## ZAKLJUČAK

Dinamička strukturalna analiza kućišta zupčaste pumpe bazirana je na principima analize izotropnih struktura sa realnom konfiguracijom i 3D elementima /5, 6, 7/. Dobijeni rezultati predstavljaju realno ponašanje razmatrane konstrukcije za odgovarajuće konturne uslove. Ekstrapolacijom modela dinamičke analize kućišta došlo se do pogodnog modela kompletne pumpe. Rezultati optimizacije i kompletne analize konstrukcije pumpe /5, 6/, treba da budu predmet daljih studija.

Razvijeni model konstrukcije pruža mogućnost uključivanja korektivnih faktora u analizu konstrukcije pumpe i njenih delova, u skladu sa mogućim odstupanjima (posebno je interesantno saopštiti dopustiva odstupanja i oštećenja u eksploataciji uređaja) geometrijske i materijalne konfiguracije predmetne konstrukcije /6, 7/.

The dynamic structural housing analysis of concern (as a representative model of hydraulic devices) shows convenience in applicability for solving problems in various structures that are met in hydraulic systems and installations.

## CONCLUSION

Dynamic structural analysis of the gear pump housing is based on the principles of analysis of isotropic structures with real geometric configurations and 3D elements /5, 6, 7/. The obtained results have shown the real behaviour of the considered structure for corresponding boundary conditions. The extrapolated model of dynamic analysis of the housing has led to a favourable solution of the pump as a whole. Results of optimization and structural analysis of the pump /5, 6/, however, are to be the subject of further studies.

The developed structural model allows for the possibility in implementing correction factors in the structural analysis of the pump and its parts with respect to possible deviations (of particular interest is to report the allowed deviations and damages in component operation) in geometry and material configuration of the structure, /6, 7/.

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