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ELASTO-PLASTIČNI PRELAZNI NAPONI U IZOTROPNOM DISKU PROMENLJIVE DEBLJINE IZLOŽENOM UNUTRAŠNJEM PRITISKU

ELASTIC-PLASTIC TRANSITION STRESESS IN AN ISOTROPIC DISC HAVING VARIABLE THICKNESS SUBJECTED TO INTERNAL PRESSURE

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Ključne reči

- izotropni disk
- pritisak
- debljina
- elastično
- plastično
- stišljivost
- prelazni napon
- tečenje
- unutrašnja plastična zona
- spoljnja plastična zona

Izvod

Elastično plastični naponi su izvedeni uz primenu teorije prelaza po Setu za izotropni disk promenljive debljine, izložen pritisku u otvoru. Dobijeni rezultati su analiziran numerički i prikazani grafički. Disk promenljive debljine od stišljivog materijala popušta pri nekom poluprečniku R_1 pri većem pritisku, u poređenju sa diskom od nestišljivog materijala koji popušta na spoljnoj površini. Ravan disk od nestišljivog materijala popušta na unutrašnjoj površini pri većem pritisku nego disk od stišljivog materijala. Obimski napon je maksimalan na spoljnjoj površini diska promenljive debljine.

Keywords

- isotropic disc
- pressure
- thickness
- elastic
- plastic
- compressibility
- transitional stress
- yielding
- inner plastic zone
- outer plastic zone

Abstract

Elastic-plastic stresses have been obtained using Seth's transition theory for an isotropic disc of variable thickness exposed to internal pressure. Results obtained are analysed numerically and depicted graphically. Disc of variable thickness made of compressible material yields at some radius R_1 at a higher pressure as compared to disc made of incompressible material that yields at the outer surface. Flat disc made of incompressible material yields on internal surface at higher pressure as compared to disc made of compressible material. Circumferential stress is maximum at the outer surface of the disc having variable thickness.

UVOD

Disk promenljive debljine se često koristi u mašinskim konstrukcijama. Okrugli prstenasti disk pod dejstvom spoljnjeg pritiska u otvoru je bio predmet istraživanja u nekoliko radova, /1-3/. Pregled literature pokazuje da je u više radova analiziran kružni prstenasti disk od materijala konstantnih osobina u različitim uslovima, /2/. Durban /4/ je izveo tačno rešenje za elasto-plastičnu prstenastu ploču sa ojačavanjem, izloženu pritisku u otvoru. Čauduri /5/ je sračunao napone u nehomogenom obrtnom prstenu menjajući Poasonov koeficijent materijala. U analizi problema ovi autori su koristili pretpostavke uprošćenja.

Prvo, deformacija je dovoljno mala da može da se koristi infinitezimalna teorija deformacija. Drugo, uprošćenja se odnose na konstitutivne jednačine za nestišljivi materijal i na kriterijum tečenja.

INTRODUCTION

Disc of variable thickness is applied frequently in mechanical engineering. Circular annular disks exposed to external pressure in the hole have been investigated in several papers, /1-3/. A literature survey indicates that in several papers circular annular disc with constant material properties has been analysed in various conditions /2/. Durban /4/ derived an exact solution for in the bore pressurized elastic-plastic, strain-hardening, annular plate. Chaudhuri /5/ calculated stresses in a non-homogeneous rotating annulus by varying Poisson's ratio of the material. In analysing the problem, these authors used simple assumptions.

First, the deformation is small enough to make infinitesimal strain theory applicable. Second, simplifications were made regarding the constitutive equations of the material like material incompressibility and an yield criterion.

Nestišljivost materijala je jedna od najvažnijih pretpostavki kojom se uprošćava problem. U stvari, u većini slučajeva nije moguće naći rešenje u zatvorenom obliku bez ove pretpostavke. Setovo prelazno rešenje ne zahteva ova uprošćenja i na taj način postavlja i rešava mnogo opštiji problem, što omogućava da se obrade ovakvi slučajevi. Teorija prelaza po Setu koristi koncept generalizovane mere deformacije i asimptotsko rešenje u kritičnim tačkama ili u prelaznim tačkama u diferencijalnim jednačinama koje definišu polje deformacija, pa se uspešno primenjuju za rešavanje većeg broja problema, /6-15, 17/. Set /6/ je definisao meru generalizovane glavne deformacije u obliku:

Incompressibility of the material is one of the most important assumptions which simplifies the problem. In fact, in most cases it is not possible to find a solution in closed form without this assumption. Seth’s transition does not require these assumptions and thus poses and solves a more general problem. Transition theory according to Seth utilizes the concept of generalised strain measure and asymptotic solution at critical points or turning points of the differential equations defining the strain field, so it has been successfully applied to a great number of problems, /6-15, 17/. Seth /6/ has defined the generalised principal strain measure as:

$$e_{ii} = \int_{0}^{\overset{A}{e}_{ii}} \left[1-2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ij} = \frac{1}{n}\left[1-\left(1-2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right]; \quad (i=1,2,3) \tag{1}$$

gde je n mera, a $\overset{A}{e}_{ii}$ je glavna komponenta Almansijeve konačne deformacije. Za $n = -2; -1; 0; 1; 2$ ona predstavlja meru po Košiju, Grin-Henkiju, Svaingeru i Almansiju, respektivno.

U ovom radu je analiziran elasto-plastični prelaz u izotropnom prstenastom disku promenljive debljine (sl. 1) izloženom unutrašnjem pritisku. Pretpostavljeno je da se debljina diska menja duž poluprečnika po zakonu

where n is the measure and $\overset{A}{e}_{ii}$ are the principal Almansi finite strain component. For $n = -2; -1; 0; 1; 2$ it gives Cauchy, Green-Hencky, Swainger and Almansi measures, respectively.

This paper has analyzed the elastic-plastic transition in an isotropic annular disc of variable thickness (Fig. 1), subjected to internal pressure. The thickness of the disc is assumed to vary along the radius obeying the law

$$h = h_0 \left(r/b\right)^{-k} \tag{2}$$

gde je h_0 konstantna debljina za $r = b$, a k je parametar debljine. Dobijeni rezultati su analizirani numerički i prikazani grafički.

where h_0 is the constant thickness at $r = b$, and k is the thickness parameter. Results obtained have been discussed numerically and depicted graphically.

OSNOVNE JEDNAČINE

Razmatra se izotropni tanki prstenasti disk promenljive debljine unutrašnjeg poluprečnika a i spoljnjeg poluprečnika b, izložen unutrašnjem pritisku p, kako je prikazano na sl. 1. Pretpostavljeno je da je disk dovoljno mali da se uspostavi ravno stanje napona, tako da je aksijalni napon T_{zz} jednak nuli. Komponente pomeranja u polarno-cilindričnim koordinatama su date jednačinama /6/:

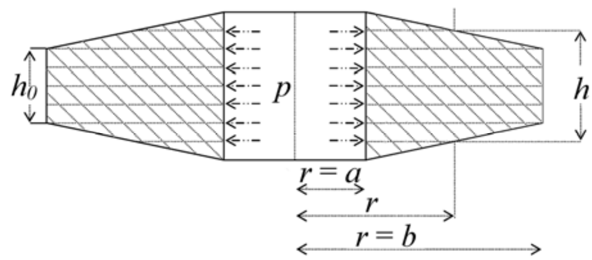
GOVERNING EQUATIONS

Consider an isotropic thin annular disc of variable thickness with internal radius a and external radius b, subjected to internal pressure p, as presented in Fig. 1. The disc is taken to be sufficiently small for establishing a state of plane stress, that is, the axial stress T_{zz} is zero. The displacement components in cylindrical polar co-ordinates are given by equations /6/:

$$u = r\left(1-\beta\right); \quad v = 0; \quad w = dz \tag{3}$$

gde je β funkcija od $r = \sqrt{x^2+y^2}$, a d je konstanta.

where β is a function of $r = \sqrt{x^2+y^2}$ and d is a constant.



Slika 1. Izotropni prstenasti disk promenljive debljine izložen pritisku u otvoru

Figure 1. Isotropic annular disc of variable thickness subjected to pressure in the hole.

Komponente konačne deformacije su date kao /6/:

The finite strain components are given by /6/:

$$\overset{A}{e}_{rr} = \frac{1}{2}\left[1-\left(r\beta'+\beta\right)^2\right]; \quad \overset{A}{e}_{\theta\theta} = \frac{1}{2}\left[1-\beta^2\right]; \quad \overset{A}{e}_{zz} = \frac{1}{2}\left[1-\left(1-d\right)^2\right]; \quad \overset{A}{e}_{r\theta} = \overset{A}{e}_{\theta z} = \overset{A}{e}_{zr} = 0 \tag{4}$$

Zamenom jed. (4) u jed. (1), generalizovane komponente deformacije su dobijene u obliku:

Substituting Eqs. (4) into Eqs. (1), the generalised components of strain are obtained in the form

$$e_{rr}=\frac{1}{n}\left[1-\left(r\beta'+\beta\right)^n\right];\quad e_{\theta\theta}=\frac{1}{n}\left[1-\beta^n\right];\quad e_{zz}=\frac{1}{n}\left[1-\left(1-d\right)^n\right];\quad e_{r\theta}=e_{\theta z}=e_{zr}=0 \tag{5}$$

gde je $\beta'=d\beta/dr$.

where $\beta'=d\beta/dr$.

Zavisnost napona i deformacija za izotropnu sredinu je /16/:

The stress-strain relation for isotropic media is /16/:

$$T_{ij}=\lambda\delta_{ij}I_1+2\mu e_{ij};\quad (i,j=1,2,3) \tag{6}$$

gde su T_{ij} i e_{ij} komponente napona i deformacija, λ i μ su Lameove konstante, $I_1 = e_{kk}$ je prva invarijanta deformacije, δ_{ij} je Kronekerova delta simbol.

Jednačine (6) za ovaj problem postaju:

where T_{ij} and e_{ij} are the stress and strain components, λ and μ are Lame's constants, $I_1 = e_{kk}$ is the first strain invariant, δ_{ij} is the Kronecker's delta symbol.

Equations (6) for this problem become:

$$T_{rr}=\frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2\mu e_{rr};\quad T_{\theta\theta}=\frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2\mu e_{\theta\theta};\quad T_{r\theta}=T_{\theta z}=T_{zr}=T_{zz}=0 \tag{7}$$

Zamenom jed. (5) u jed. (7), naponi prelaze u:

Substituting Eqs. (5) into Eqs. (7), the stresses become:

$$T_{rr}=\frac{2\mu}{n}\left[3-2c-\beta^n\left\{1-c+\left(2-c\right)\left(\frac{r\beta'}{\beta}+1\right)^n\right\}\right];T_{\theta\theta}=\frac{2\mu}{n}\left[3-2c-\beta^n\left\{2-c+\left(1-c\right)\left(\frac{r\beta'}{\beta}+1\right)^n\right\}\right];T_{r\theta}=T_{\theta z}=T_{zr}=T_{zz}=0 \tag{8}$$

gde je c faktor stišljivosti materijala, koji zavisi od Lameovih konstanti, dat u obliku

where c, compressibility factor of the material in term of Lame's constant, is given in the form

$$c=\frac{2\mu}{\lambda+2\mu}$$

Sve jednačine ravnoteže su zadovoljene, izuzev:

All the equations of equilibrium are satisfied, except:

$$\frac{d}{dr}\left(rT_{rr}h\right)-hT_{\theta\theta}=0 \tag{9}$$

Zamenom jed. (8) u jed. (9), nelinearna diferencijalna jednačina po β se dobija kao:

Using Eqs. (8) in Eq. (9), a non-linear differential equation in β is obtained as:

$$(2-c)n\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta}=\left[\frac{rh'}{h}\left[3-2c-\beta^n\left\{1-c+(2-c)(P+1)^n\right\}\right]+\beta^n\left[1-(P+1)^n\right]-np\beta^n\left[1-c+(2-c)(P+1)^n\right]\right]\Bigg\} \tag{10}$$

gde je $h'=\frac{dh}{dr}$ i $r\beta'=\beta P$ (P funkcija od β, a β je funkcija od r).

Iz jed. (10), prelazne tačke za β su $P = -1$ i $P = \pm\infty$. Granični uslovi su:

where $h'=\frac{dh}{dr}$ and $r\beta'=\beta P$ (P is function of β and β is function of r).

From Eq. (10), the turning points of β are $P = -1$ and $P = \pm\infty$. The boundary conditions are:

$$T_{rr}=-p\ \text{ at }\ r=a\ \text{ and }\ T_{rr}=0\ \text{ at }\ r=b. \tag{11}$$

REŠENJE PREKO GLAVNIH NAPONA

Za određivanje plastičnih napona, funkcija prelaza je uzeta preko glavnih napona (Set /7, 8/, Hulsurkar /9/ i Pankaj /10-15/) u tački prelaza $P\to\pm\infty$. Funkcija prelaza R^* uzima se kao:

SOLUTION THROUGH THE PRINCIPAL STRESSES

For finding the plastic stress, the transition function is taken through the principal stress (Seth /7, 8/, Hulsurkar /9/ and Pankaj /10-15/) at the transition point $P\to\pm\infty$. The transition function R^* is taken as:

$$R^*=T_{\theta\theta}=\frac{2\mu\beta^n}{n}\left[3-2c-\beta^n\left\{2-c+\left(1-c\right)\left(1+P\right)^n\right\}\right] \tag{12}$$

Logaritamskim diferenciranjem jed. (12) u odnosu na r dobija se:

Taking the logarithmic differentiation of Eq. (12) with respect to r, one gets:

$$\frac{d(\log R^*)}{dr}=\frac{nP}{r\left[1-\left(P+1\right)^n\right]}\left[1-\left(P+1\right)^n-\beta\left(P+1\right)^{n-1}\frac{dP}{d\beta}\right] \tag{13}$$

Zamenom $dP/d\beta$ vrednosti iz jed. (10) u jed. (13) je

Substituting $dP/d\beta$ value from Eq. (10) in Eq. (13) it is

$$\frac{d(\log R^*)}{dr}=\frac{2\mu}{rnR}\left[\frac{-rh'}{h}\left(\frac{1-c}{2-c}\right)\left\{3-2c-\beta^n\left[1-c+(2-c)(1+P)^n\right]\right\}-\left(\frac{1-c}{2-c}\right)\beta^n\left[1-(P+1)^n\right]+n\beta^nP\left(\frac{2c-3}{2-c}\right)\right] \tag{14}$$

Uzimanjem asimptotske vrednosti $P \to \pm\infty$ i korišćenjem jed. (2), posle integracije dobija se:

Taking asymptotic value as $P \to \pm\infty$ and using Eq. (2) one gets after integration:

$$R^* = T_{\theta\theta} = \frac{A r^{\frac{-1}{(2-c)}}}{h} \tag{15}$$

gde je A konstanta integracije.

Zamenom jed. (15) u jed. (9), posle integracije je:

where A is a constant of integration.

Substituting Eq. (15) into (9), after integration one gets:

$$rhT_{rr} = A\left(\frac{2-c}{1-c}\right) r^{\left(\frac{1-c}{2-c}\right)} + B \tag{16}$$

gde je B konstanta integracije.

Zamenom jed. (11) u jed. (16) dobija se:

where B is a constant of integration.

Substituting Eq. (11) in Eq. (16), we obtain:

$$A = \frac{pah(a)(1-c)}{(2-c)\left[b^{\left(\frac{1-c}{2-c}\right)} - a^{\left(\frac{1-c}{2-c}\right)}\right]}; \quad B = \frac{-pah(a) b^{\left(\frac{1-c}{2-c}\right)}}{\left[b^{\left(\frac{1-c}{2-c}\right)} - a^{\left(\frac{1-c}{2-c}\right)}\right]}.$$

Zamenom vrednosti A i B u jed. (15) i (16) dobijaju se naponi prelaza kao:

Substituting the value of A and B in Eqs. (15) and (16), we get the transitional stresses as:

$$T_{rr} = pR_0^{1-k} R^{k-1}\left[\frac{R^{\left(\frac{1-c}{2-c}\right)} - 1}{1 - R_0^{\left(\frac{1-c}{2-c}\right)}}\right] \tag{17}$$

$$T_{\theta\theta} = pR_0^{1-k}\left(\frac{1-c}{2-c}\right)\frac{R^{\left[\left(\frac{1-c}{2-c}\right)+k-1\right]}}{\left[1 - R_0^{\left(\frac{1-c}{2-c}\right)}\right]} \tag{18}$$

gde je $R = r/b$ i $R_0 = a/b$.

Iz jed. (17) i (18) se dobija:

where $R = r/b$ and $R_0 = a/b$.

From Eqs. (17) and (18), one gets:

$$T_{\theta\theta} - T_{rr} = \frac{pR_0^{1-k}}{\left[1 - R_0^{\left(\frac{1-c}{2-c}\right)}\right]} R^{k-1}\left\{-\frac{1}{(2-c)} R^{\frac{1-c}{2-c}} + 1\right\} \tag{19}$$

Maksimalna vrednost $|T_{\theta\theta} - T_{rr}|$ se javlja za poluprečnik $R = \left[\frac{(k-1)(2-c)^2}{k(2-c)-1}\right]^{(2-c/1-c)} = R_1$, koji zavisi od vrednosti k i c. Na primer, ako se uzme c = 0; 0.25; 0.5 tečenje počinje na unutrašnjoj površini za vrednosti k = 1.273459; 1.316213; 1.374582, respektivno, a za vrednosti k = 1.5; 1.571429; 1.666667 tečenje počinje na spoljnjoj površini (tab. 1). Za tečenje na $R = R_1$, jed. (19) prelazi u

The maximum value of $|T_{\theta\theta} - T_{rr}|$ occurs (say) at radius $R = \left[\frac{(k-1)(2-c)^2}{k(2-c)-1}\right]^{(2-c/1-c)} = R_1$, which depends upon the values of k and c. For example, taking c = 0; 0.25; 0.5 yielding starts at the internal surface for values k = 1.273459; 1.316213; 1.374582, respectively, and for values k = 1.5; 1.571429; 1.666667 yielding starts at the external surface (Table 1). For yielding at $R = R_1$, Eq. (19) becomes

$$\left|T_{\theta\theta} - T_{rr}\right|_{R=R_1} = \left|\frac{(1-c)pR_0^{1-k}\left[(k-1)(2-c)^2\right]^{\frac{(2-c)(k-1)}{(1-c)}}}{\left[k(2-c)-1\right]^{\frac{k(2-c)-1}{(1-c)}}\left[1 - R_0^{\left(\frac{1-c}{2-c}\right)}\right]}\right| \equiv Y \text{ (say) (na pr.)}. \tag{20}$$

a potrebni pritisak za tečenje je

and the required pressure for yielding is

$$p_i = \frac{p}{Y} = \frac{\left[1 - R_0^{\left(\frac{1-c}{2-c}\right)}\right]\left[k(2-c)-1\right]^{\frac{(2-c)k-1}{1-c}}}{(1-c)R_0^{1-k}\left[(k-1)(2-c)^2\right]^{\frac{(2-c)(k-1)}{(1-c)}}}. \tag{21}$$

Zamenom jed. (21) u jed. (17) i (18) dobijaju se prelazni naponi u bezdimenzionalnom obliku:

Using Eq. (21) in Eqs. (17) and (18), the transitional stresses in non-dimensional form are:

$$\sigma_r = \frac{T_{rr}}{Y} = p_1 R_0^{1-k} R^{k-1} \left[\frac{R^{\left(\frac{1-c}{2-c}\right)} - 1}{1 - R_0^{\left(\frac{1-c}{2-c}\right)}} \right] \quad (22)$$

$$\sigma_\theta = \frac{T_{\theta\theta}}{Y} = p_1 R_0^{1-k} \left(\frac{1-c}{2-c} \right) \frac{R^{\left[\left(\frac{1-c}{2-c}\right)+k-1\right]}}{\left[1 - R_0^{\left(\frac{1-c}{2-c}\right)}\right]} \quad (23)$$

Naponi stanja pune plastičnosti se dobijaju iz jed. (17) i (18) uzimajući da $c \to 0$. Postoje dve plastične zone: unutrašnja ($R_0 \le R \le R_1$) i spoljna ($R_1 \le R \le 1$).

Za unutrašnju plastičnu zonu jed. (19) postaje:

Stresses for fully-plastic state are obtained from Eqs. (17) and (18) by taking $c \to 0$. There are two plastic zones: inner ($R_0 \le R \le R_1$) and outer ($R_1 \le R \le 1$).

For inner plastic zone, Eq. (19) becomes:

$$\left| T_{\theta\theta} - T_{rr} \right|_{R=R_0} = \left| \frac{p\left(2-\sqrt{R_0}\right)}{2\left(1-\sqrt{R_0}\right)} \right| \equiv Y^* \text{ (say) (na pr.)} \quad (24)$$

i potreban pritisak za puno plastično stanje je dat kao:

and the required pressure for fully plastic state is given by:

$$p_1^* \equiv \frac{p}{Y^*} = \frac{2\left(1-\sqrt{R_0}\right)}{\left(2-\sqrt{R_0}\right)} \quad (25)$$

Zamenom jed. (25) u jed. (17) i (18) dobijaju se naponi unutrašnje plastične zone kao:

Using Eq. (25) in Eqs. (17) and (18), the stresses for the inner plastic zone are:

$$\sigma_r^* = \frac{T_{rr}}{Y^*} = \frac{p_1^* R_0^{1-k} R^{k-1}\left(\sqrt{R}-1\right)}{\left(1-\sqrt{R_0}\right)} \quad (26)$$

$$\sigma_\theta^* = \frac{T_{\theta\theta}}{Y^*} = \frac{p_1^* R_0^{1-k} R^{k-\frac{1}{2}}}{2\left(1-\sqrt{R_0}\right)} \quad (27)$$

Za spoljnju plastičnu zonu jed. (19) prelazi u:

For outer plastic zone, Eq. (19) becomes:

$$\left| T_{\theta\theta} - T_{rr} \right|_{R=1} = \left| \frac{p R_0^{1-k}}{2\left(1-\sqrt{R_0}\right)} \right| \equiv Y^{**} \text{ (say) (na pr.)} \quad (28)$$

i potreban pritisak je:

and the required pressure is:

$$p_1^{**} \equiv \frac{p}{Y^{**}} = \frac{2\left(1-\sqrt{R_0}\right)}{R_0^{1-k}} \quad (29)$$

Zamenom jed. (29) u jed. (17) i (18) dobijaju se naponi za spoljnju plastičnu zonu kao:

Using Eq. (29) in Eqs. (17) and (18), the stresses for the outer plastic zone are:

$$\sigma_r^{**} = \frac{T_{rr}}{Y^{**}} = \frac{p_1^{**} R_0^{1-k} R^{k-1}\left(\sqrt{R}-1\right)}{\left(1-\sqrt{R_0}\right)} \quad (30)$$

$$\sigma_\theta^{**} = \frac{T_{\theta\theta}}{Y^{**}} = \frac{p_1^{**} R_0^{1-k} R^{k-\frac{1}{2}}}{2\left(1-\sqrt{R_0}\right)} \quad (31)$$

Poseban slučaj

Za ravan disk (k = 0) prelazni elasto-plastični naponi (17) i (18) postaju:

Particular case

For a flat disc (k = 0) elastic-plastic transitional stresses (17) and (18) become:

$$T_{rr} = \frac{pR_0}{R} \left[\frac{R^{\left(\frac{1-c}{2-c}\right)} - 1}{1 - R_0^{\left(\frac{1-c}{2-c}\right)}} \right] \quad (32)$$

$$T_{\theta\theta} = pR_0 \left(\frac{1-c}{2-c} \right) \frac{R^{-1/(2-c)}}{\left[1 - R_0^{\left(\frac{1-c}{2-c}\right)}\right]} \quad (33)$$

Vidi se da je $|T_{\theta\theta} - T_{rr}|$ najveće na unutrašnjoj površini i da tečenje počinje na otvoru, tako da je:

It is seen that $|T_{\theta\theta} - T_{rr}|$ is maximum at the internal surface and yielding takes place at the bore, so:

$$\left| T_{\theta\theta} - T_{rr} \right|_{R=R_0} = \left| \frac{p}{\left[1 - R_0^{\left(\frac{1-c}{2-c}\right)}\right]} \left[-\frac{1}{(2-c)} R_0^{(1-c)/(2-c)} + 1 \right] \right| \equiv Y_1 \text{ (say) (na pr.)} \quad (34)$$

Pritisak P_i potreban za početno tečenje je dat sa:

The pressure P_i required for initial yielding is given by:

$$P_i = \frac{p}{Y_1} = \frac{\left[1 - R_0^{\left(\frac{1-c}{2-c}\right)}\right]}{\left[-\frac{1}{(2-c)} R_0^{(1-c)/(2-c)} + 1\right]}. \quad (35)$$

Zamenom jed. (35) u jed. (32) i (33) dobijaju se prelazni naponi u obliku:

Using Eq. (35) in Eqs.(32) and (33), the transitional stresses are:

$$\sigma_r = \frac{T_{rr}}{Y_1} = \frac{P_i R_0}{R}\left[\frac{R^{\left(\frac{1-c}{2-c}\right)} - 1}{1 - R_0^{\left(\frac{1-c}{2-c}\right)}}\right] \quad (36) \qquad \sigma_\theta = \frac{T_{\theta\theta}}{Y_1} = P_i R_0 \left(\frac{1-c}{2-c}\right)\frac{R^{-1/(2-c)}}{\left[1 - R_0^{\left(\frac{1-c}{2-c}\right)}\right]} \quad (37)$$

Za stanje potpune plastičnosti ($c \to 0$) na spoljnoj površini ($R = 1$) dobija se:

For fully-plastic state ($c \to 0$) at the external surface ($R = 1$), we have:

$$\left|T_{rr} - T_{\theta\theta}\right|_{R=1} = \left[\frac{pR_0}{2\left[1 - \sqrt{R_0}\right]}\right] = Y_1^* \quad (38)$$

a pritisak P_f potreban za potpuno plastično stanje je:

and pressure P_f required for fully plastic state is:

$$P_f \equiv \frac{p}{Y_1^*} = \frac{2\left(1 - \sqrt{R_0}\right)}{R_0}. \quad (39)$$

Zamenom jed. (39) u jed. (32) i (33) dobijaju se naponi pune plastičnosti kao:

Using Eq. (39) in Eqs.(32) and (33), we get the stresses for fully plastic state as:

$$\sigma_r = \frac{T_{rr}}{Y_1^*} = \frac{P_f R_0}{R}\left[\frac{\sqrt{R} - 1}{1 - \sqrt{R_0}}\right] \quad (40) \qquad \sigma_\theta = \frac{T_{\theta\theta}}{Y_1^*} = \frac{P_f R_0 R^{-1/2}}{2\left[1 - \sqrt{R}\right]}. \quad (41)$$

RESULTATI I DISKUSIJA

U tab. 1 date su vrednosti napona za početno tečenje i stanje pune plastičnosti za izotropni disk promenljive debljine. Tečenje se javlja na nekom poluprečniku ($R = R_1$), na unutrašnjoj ($R_1 = 0{,}5$) ili spoljnjoj površini ($R_1 = 1$), zavisno od vrednosti k i c. Na primer, tečenje se javlja na unutrašnjoj površine diska od nestišljivog materijala ($c = 0{,}25$) pri pritisku 0,4466247 za $k = 1{,}3174582$, dok je taj pritisak na spoljnjoj površini 0,4035501 za $k = 1{,}571429$. Iz tabele se takođe vidi da se u disku promenljive debljine od nestišljivog materijala javlja tečenje pri većem pritisku u poređenju sa diskom od stišljivog materijala.

U tabeli 2. su date vrednosti pritiska za početno tečenje P_i i stanje potpune plastičnosti P_f za izotropni disk promenljive debljine ($k = 1{,}5$) i za ravan disk ($k = 0$) za različite vrednosti c.

Iz tab. 2. se vidi da se kod izotropnog diska od stišljivog materijala promenljive debljine ($k = 1{,}5$) na nekom poluprečniku R_1 javlja tečenje pri većem pritisku u poređenju sa diskom od nestišljivog materijala, kod kojeg sa tečenje javlja na spoljnjoj površini, međutim, to nije slučaj sa ravnim diskom, odnosno, za ravan disk od nestišljivog materijala potreban je procentualno mnogo veći pritisak da se ostvari potpuna plastičnost u poređenju sa diskom promenljive debljine.

RESULTS AND DISCUSSION

In Table 1 pressure values required for initial yielding and fully plastic state for an isotropic disc of variable thickness are given. Yielding occurs at any radius ($R = R_1$), or at the internal ($R_1 = 0.5$) or external surface ($R_1 = 1$) depending on k and c values. For example, yielding occurs at the internal surface of the disc made of compressible material ($c = 0.25$) at a pressure 0.4466247 for $k = 1.3174582$, whereas at the outer surface at a pressure 0.4035501 for $k = 1.571429$. It is also seen from table that the disc of variable thickness and made of incompressible material yields at a higher pressure as compared to disc made of compressible material.

In Table 2 pressure values are given for initial yielding P_i and fully plastic state P_f for an isotropic disc of variable thickness ($k = 1.5$) and flat disc ($k = 0$) for different c values.

It can be seen from Table 2 that an isotropic disc made of compressible material with variable thickness ($k = 1.5$) at some radius R_1 yields at a higher pressure as compared to disc made of incompressible material which yields at the outer surface, but this is not the case with a flat disc, that is, the flat disc of incompressible material requires a much higher percentage increase in pressure to become fully plastic as compared to the disc of variable thickness.

Tabela 1. Pritisak za početno tečenje (p_i) i stanje potpune plastičnosti (p_f) izotropnog diska promenljive debljine za različite vrednosti k i c
Table 1. Pressure for initial yielding (p_i) and fully plastic state (p_f) for an isotropic disc of variable thickness for different values k and c.

c	k	Tečenje se javlja za	Izotropni disk promenljive debljine $\left[h=h_0(r/b)^{-k}\right]$ $r=a$	$r=b$	Pritisak potreban za tečenje p_i	Pritisak potreban za potpunu plastičnost p_f	Procentualni porast pritiska od inicijalnog tečenja do potpune plastičnosti $P=\left(\frac{p_f}{p_i}-1\right)\times 100$
c	k	Yielding initiates at	Isotropic disc of variable thickness $\left[h=h_0(r/b)^{-k}\right]$ $r=a$	$r=b$	Pressure for initial yielding p_i	Pressure for fully-plastic state p_f	Pressure increase from initial yielding to fully plastic state $P=\left(\frac{p_f}{p_i}-1\right)\times 100$
0	1.273459	$R_1=0.5$	$h=h_0\,(2.417405)$	$h=h_0$	0.4530818	0.484640742	6.965386786 %
0.25	1.316213		$h=h_0\,(2.490116)$	$h=h_0$	0.4466273	0.47048926	5.342693651 %
0.5	1.374582		$h=h_0\,(2.592927)$	$h=h_0$	0.4381275	0.451833986	3.128427988 %
0	1.316058	$R_1=0.6$	$h=h_0\,(2.489848)$	$h=h_0$	0.4512678	0.470539811	4.270626831 %
0.25	1.363723		$h=h_0\,(2.573484)$	$h=h_0$	0.4446324	0.45524772	2.387442092 %
0.5	1.428203		$h=h_0\,(2.691113)$	$h=h_0$	0.4359177	0.453081839	3.93748271 %
0	1.381854	$R_1=0.75$	$h=h_0\,(2.606031)$	$h=h_0$	0.4424812	0.453081839	2.395723618 %
0.25	1.437485		$h=h_0\,(2.708483)$	$h=h_0$	0.4349341	0.453081839	4.172531641 %
0.5	1.512061		$h=h_0\,(2.852173)$	$h=h_0$	0.4251179	0.453081839	6.57791428 %
0	1.427605	$R_1=0.85$	$h=h_0\,(2.689997)$	$h=h_0$	0.4381106	0.453081839	3.417219549 %
0.25	1.489106		$h=h_0\,(2.80715)$	$h=h_0$	0.4300967	0.453081839	5.344189641 %
0.5	1.571265		$h=h_0\,(2.971651)$	$h=h_0$	0.4134543	0.453081839	9.584492404 %
0	1.5	$R_1=1$	$h=h_0\,(2.828427)$	$h=h_0$	0.4142136	0.453081839	9.383632136 %
0.25	1.571429		$h=h_0\,(2.971989)$	$h=h_0$	0.4035501	0.453081839	12.27399866 %
0.5	1.666667		$h=h_0\,(3.174802)$	$h=h_0$	0.3898815	0.453081839	16.21014514 %

Tabela 2. Pritisak za početno tečenje (p_i) i stanje pune plastičnosti (p_f) izotropnog diska promenljive debljine (k = 1,5) i ravnog diska (k = 0) za različite vrednosti c
Table 2. Pressure for initial yielding (p_i) and fully plastic state (p_f) for an isotropic disc of variable thickness (k = 1.5) and flat disc (k = 0) for different c values.

c	k	Tečenje se javlja za	Izotropni disk promenljive debljine $\left[h=h_0(r/b)^{-k}\right]$ $r=a$	$r=b$	Pritisak potreban za tečenje p_i	Pritisak potreban za potpunu plastičnost p_f	Procentualni porast pritiska od inicijalnog tečenja do potpune plastičnosti $P=\left(\frac{p_f}{p_i}-1\right)\times 100$
c	k	Yielding occurs at	Isotropic disc having variable thickness $\left[h=h_0(r/b)^{-k}\right]$ $r=a$	$r=b$	Pressure for initial yielding p_i	Pressure for fully-plastic state p_f	Pressure increase from initial yielding to fully plastic state $P=\left(\frac{p_f}{p_i}-1\right)\times 100$
0	1.5	$R_1=1$	$h=h_0\,(2.828427)$	$h=h_0$	0.4142136	0.45308189	9.383632136 %
0.25	1.5	$R_1=0.974854$	$h=h_0\,(2.828427)$	$h=h_0$	0.4220108	0.45308189	7.362626636 %
0.5	1.5	$R_1=0.965489$	$h=h_0\,(2.828427)$	$h=h_0$	0.4271293	0.45308189	6.076037342 %
0	0.0000	$R_0=0.5$	$h=h_0$	$h=h_0$	0.453082	1.1716	158.5786 %
0.25	0.0000		$h=h_0$	$h=h_0$	0.446627	1.1716	162.3155 %
0.5	0.0000		$h=h_0$	$h=h_0$	0.435243	1.1716	169.1764 %

Na sl. 2 su nacrtane krive zavisnosti napona i odnosa poluprečnika r/b za stanje potpune plastičnosti. Obimski napon je najveći na unutrašnjoj površini ravnog diska, ali je kod diska promenljive debljine najveći na spoljnjoj površini.

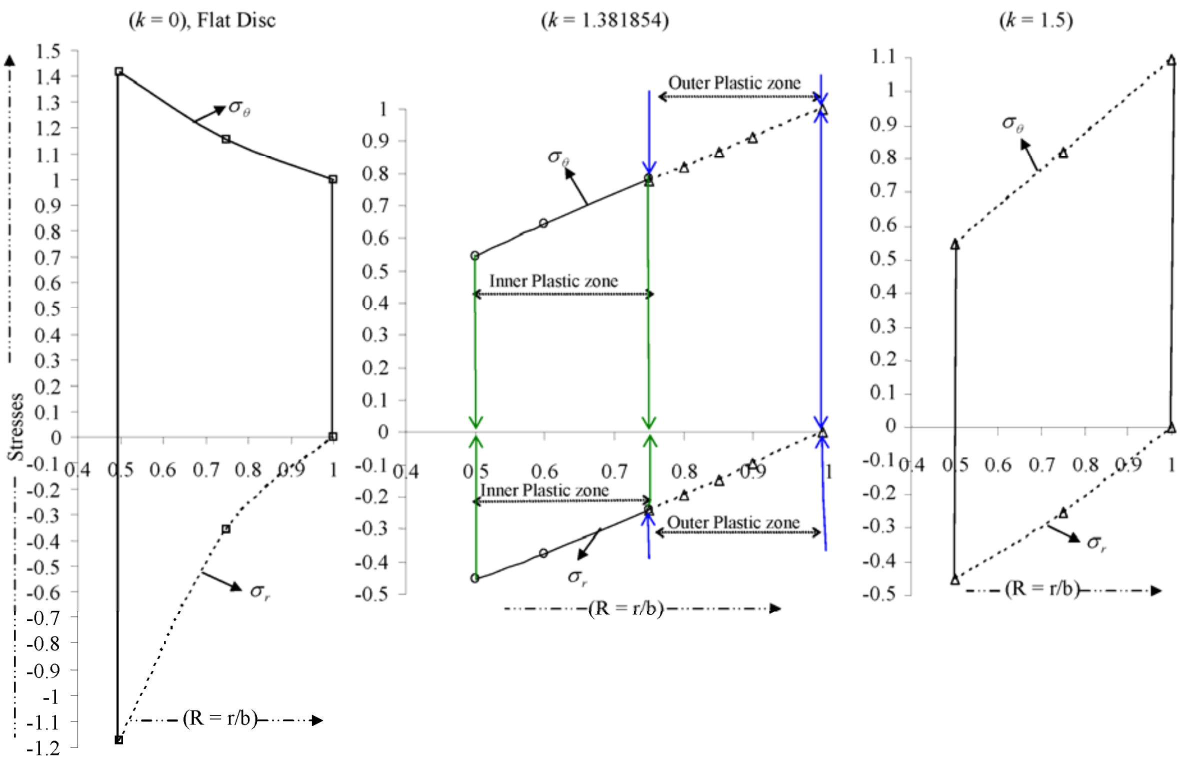
ZAKLJUČAK

Disk od stišljivog materijala promenljive debljine teče pri nekom poluprečniku R_1 pri višem pritisku u poređenju sa diskom od nestišljivog materijala koji teče na spoljnjoj površini. Ravan disk od nestišljivog materijala teče na unutrašnjoj površini pri višem pritisku u poređenju sa diskom od stišljivog materijala. Obimski napon je najveći na spoljnjoj površini diska promenljive debljine.

In Fig. 2, curves are drawn for stresses and radii ratio r/b for fully-plastic state. Circumferential stress is maximum at the internal surface of the flat disc, but it is maximum at the outer surface of the disc having variable thickness.

CONCLUSIONS

Disc of compressible material having variable thickness yields at some radius R_1 at a higher pressure compared to disc of incompressible material which yields at the outer surface. Flat disc made of incompressible material yields at internal surface at higher pressure as compared to disc of compressible material. Circumferential stress is maximum at the outer surface of the disc having variable thickness.



Slika 2. Naponi stanja potpune plastičnosti za različite vrednosti *k* zavisno od odnosa poluprečnika *R* = *r*/*b*
Figure 2. Stresses at fully-plastic state for different values of *k* with respect to radii ratio *R* = *r*/*b*.

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