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MODEL VEROVATNOĆE PITING KOROZIJE I PROCENA ZAMORNOG VEKA TURBINSKIH LOPATICA

PROBABILISTIC MODEL FOR PITTING CORROSION AND FATIGUE LIFE ESTIMATION OF TURBINE BLADES

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Ključne reči

- korozija, piting
- zamor, rast prsline, oštećenje
- predviđanje radnog veka
- verovatnoća, pouzdanost

Izvod

Radni uslovi lopatica turbina su veoma složeni zbog mehaničkih opterećenja i nepovoljnog, i pod određenim uslovima, koroziono aktivnog radnog fluida. Mehanička opterećenja su posledica varijacija u pritisku i temperaturi pri startovanju i isključenju postrojenja i mogu da utiču na smanjenje dinamičkih karakteristika rotorskih lopatica.

Različiti korozioni mehanizmi mogu da se razviju kod turbinskih lopatica u zavisnosti od radne sredine. Međutim, u najnovijim studijama posvećenim ovom problemu je ipak pokazano da je dominantan mehanizam razaranja turbinskih lopatica posledica širenja zamorne prsline koja je inicirana na piting korozionom defektu. Ovo je veoma kompleksan elektrohemijski i mehanički mehanizam razaranja.

Da bi se sprečila pojava neželjenih razaranja, današnja istraživanja su usmerena na definisanje procedure u cilju procene veka i razvoja oštećenja u proučavanim konstrukcionim delovima, kao i na predviđanje optimalnog vremena njihove kontrole konvencionalnim postupcima. Uzimajući u obzir veoma veliku disperziju relevantnih parametara kao i nepostojanje dobro definisanog modela do danas, kojim bi bilo opisano oštećivanje, model koji je primenjen u ovom radu je verovatnosni model tolerancije oštećenja.

U modelu je pretpostavljeno da je većina parametara statističkog karaktera i da na osnovu toga može da se odredi najbolja procena verovatnoće pojave razaranja. Razmatrani model se konceptualno sastoji od sedam stadijuma: inicijacija defekta iz kojeg će se obrazovati pit, rast pita, inicijacija kratke prsline na pitu, rast kratke prsline, prelaz od rasta kratke prsline u rast duge prsline, rast duge prsline i konačni lom. Ovaj kompleksni model je u saglasnosti sa nekoliko, još uvek nepotpuno definisanih, fizičkih koncepata i nekih dokazanih pojava. Primenom verovatnoće predloženi mehanicistički model daje verovatnoću razaranja koja je u skladu sa rezultatima klasičnih determinističkih modela.

Keywords

- corrosion, pitting
- · fatigue, crack growth, damage
- life prediction
- probability, reliability

Abstract

Operating conditions of turbine blades are very complicated because of the mechanical loads and severe environmental conditions of corrosively active fluid. Pressure and temperature cause mechanical load variations at start-up and shut-down, and may deteriorate dynamic properties of rotor blades.

Various corrosion mechanisms could develop on turbine blades depending on the operational environment. However, several updated studies of this problem show that the dominant failure mechanism in turbine blades results from fatigue crack propagation, initiating on pitting corrosion defects. This is a complex electrochemical and mechanical failure mechanism.

In order to prevent the occurrence of the unacceptable damage, today's research is aimed at defining a procedure for estimating the lifetime and development of damage in studied structural parts, and in scheduling optimal periods for inspection by conventional methods. Considering the very large dispersion of relevant parameters and the lack of well defined models that exactly define the damaging process, the model developed here is a probabilistic damage tolerance model.

The model takes into account that most parameters are likely statistical, and based on this it attempts to determine the best probabilistic evaluation of the occurring failure. The concept of the studied model consists of seven stages: initiation of defect that creates the pit; pit growth; short crack initiation from pit; short crack growth; transition from short to long crack growth; long crack growth; and final fracture. This complex model coincides with several, not yet well defined, physical concepts and some proven events. By applying probability, the suggested mechanistic model estimates the probability of failure that abides by the results of classical deterministic models.

UVOD

Parne turbine, kao i gasne turbine, su toplotne mašine, koje pretvaraju toplotnu u mehaničku energiju i koriste se u industriji uglavnom u sledeće svrhe: za proizvodnju električne energije pokretanjem električnih generatora i za pokretanje opreme kao što su kompresori, ventilatori i pumpe. Funkcionalno, ova oprema spada u kritičnu bez obzira na vrstu postrojenja. Da bi rad bilo kog industrijskog postrojenja bio uspešan, neophodno je da se vitalne komponente odlikuju velikom pouzdanošću u radu. Lopatice turbina su komponente koje obezbeđuju tu konverziju energije. Pri transformaciji kinetičke energije fluida, gasa ili pare, u mehaničku energiju rotacijom vratila, ove komponente su izložene različitim uslovima opterećenja. Naime, lopatice turbina su podvrgnute varijacijama pritiska, temperature, delovanju korozione sredine i mogu da budu osetljive na dinamičke uslove.

Stoga su lopatice izložene delovanju kompleksnih mehanizama oštećivanja, s tim što oštećenja tokom vremena rada rastu dovodeći do gubitka radne sposobnosti. Mehanizmi oštećivanja odgovorni za lom lopatica su istraživani u nekoliko radova, /1-5/, sa aspekta makro i mikro karakteristika prelomnih površina, pri čemu je mehanizam zamora sa inicijalnim pitingom istaknut u ovim radovima. U cilju poboljšanja radnih uslova i prevencije loma, istraživači su težili da otkriju put za razumevanje i određivanje stanja komponente, kao i razvoj nastalih oštećenja, /6, 7/. Prema najnovijim literaturnim podacima i standardima, /8/, neki istraživači su postigli taj cilj primenom probabilističkog pristupa. Oni su uzeli u obzir nejasnoće, neizvesnost i slučajni karakter vrednosti i parametara koji su korišćeni u modelima procene. U ovaj probabilistički okvir su implementirana dva glavna filozofska koncepta: frekventni i Bajesov pristup, /9/. Frekventni pristup, kao objektivan probabilistički pristup, se uglavnom koristi zbog svoje osobine da je matematički lako razumljiv i lako primenljiv, /9, 10/.

Siguran radni vek lopatica je određen u radu koji je dao Naim, /10/, pri čemu je uzeo u obzir funkcionalne neizvesnosti koje se javljaju tokom procene rada avionskih motora. Na osnovu konstatacije da granično stanje, saglasno promenljivoj raspodeli pritiska i temperature ima za posledicu visoko opterećenje, on je odredio siguran radni vek lopatica u uslovima postojanja oštećenja od puzanja i zamora. U radu /11/ je primenjena ista procedura za određivanje sigurnog radnog veka lopatica visokog pritiska turbo reaktora determinističkim pristupom.

Ši i Mahadevan, /12/, su istraživali verovatnoću pojave piting korozije i zamora i određivali radni vek primenom modela tolerancije oštećenja. Oni su primenili sedmostepeni probabilistički model za zamornu prslinu iniciranu u pitu, koji su identifikovali Gosvani i Hoper, /14/, primenom osnovnog Parisovog modela za rast prsline i Faradejevog zakona za elektrohemijsku interakciju između metala i njegovog okruženja. Ovaj model su intenzivno istraživali i razvili Harlou i Vei, /15, 18/. Mehanizam implementiran u njihovoj studiji je razvoj oštećenja od elektrohemijske inicijacije pita do krtog loma, kroz stadijume obrazovanja kratke prsline, širenja kratke prsline i konačno kroz širenje duge prsline. Koraci u razvoju oštećenja i koegzistiranje dva

INTRODUCTION

Steam turbines, as also gas turbines, are classified as heat engines that transfer heat into mechanical energy and are generally used in the industry for several purposes: providing power for driving electric generators and equipment such as compressors, fans and pumps. These are the usual potentially critical components in many industrial fields. Efficiency of an industrial installation requires that vital components operate with high reliability. Energy is converted by turbine blades. These elements are subjected to various loading conditions when transforming the fluid kinetic energy of the gas or steam into mechanical energy through the rotation of the shaft. Thus, turbine blades are exposed to pressure and temperature variations, and are influenced by the corrosive environment, and may be susceptible to dynamical conditions.

Hence, blades are subjected to complex mechanisms of damage, progressing in exploitation, leading to the loss of working capacity. Damage mechanisms responsible for blade ruptures have been researched in several papers, /1-5/, regarding macro- and microscopic aspects of fractured surfaces, whereby a fatigue mechanism with corrosion pit initiation is observed in them. In order to enhance operating conditions and prevent failure, researchers are looking for ways to understand and estimate the state of the component and its damage evolution, /6, 7/. According to the latest world standards, /8/, some researchers have achieved this knowledge by applying probabilistic approaches. They take into account the vagueness, uncertainties and randomness of values and parameters used in estimating models. Here, the probabilistic framework uses two principal philosophical concepts: frequentist and Bayesian approaches, /9/. The frequentist approach, as an objective probabilistic approach, is the most widely used, due to its capability of being mathematically implemented and widely understood and easily computed, /9-10/.

Safe lifetime of blades are estimated in Naeem's et al article, /10/, while taking into account functional uncertainties during the evaluation of flight engines. Based on the fact that the limit state, according to the varying distribution of pressure and temperature, results in high loads, he estimated the safe lifetime of blades to fatigue and creep damage. In article /11/, the same procedure is used to define safe lifetime of reaction turbine high pressure blades by a deterministic approach.

Shi and Mahadevan, /12/, studied complex probabilistic pitting corrosion and fatigue, and estimated the lifetime through damage tolerance procedures. They used the seven-stage probabilistic model for a fatigue crack initiated by pitting, identified by Goswani and Hopper, /14/, using the basic Paris model for crack growth and Faraday's law for electrochemical interaction between the metal and its environment. This model was intensively studied and developed by Harlow and Wei, /15, 18/. The mechanism involved in their study is the evolution of damage from the electrochemical corrosion pit initiation to brittle fracture, where damage develops from short crack formation to short crack propagation and finally, to long crack propagation. Damage evolution steps and coexistence of two simultaneous

procesa oštećivanja istovremeno, korozije i zamora, imaju sopstvenu varijabilnost. Što se tiče hemijskih i mehaničkih osobina, one ne mogu tačno da se odrede u determinističkom smislu. Pokazano je da je, uzimajući u obzir aspekte verovatnoće u kombinaciji sa modelom mehaničke degradacije, moguće izvršiti procenu procesa oštećivanja i radnog veka. U studijama o zamoru uobičajeno se razmatraju dva aspekta: 1. sigurni radni vek u kojem se, hipotetički, neće inicirati oštećenja i rast prsline, i 2. koji je posledica rasta tog oštećenja u uslovima promenljivog opterećenja.

U ovom radu su prikazane sve promenljive za koje je moguća međusobna interakcija na oštećenju na površini komponente i njihova promenljivost koja se ogleda kroz podatke saopštene u literaturi, i u tom smislu faze u zamornom veku komponente. Takođe, prikazan je i model verovatnoće kojim su obuhvaćene slučajnosti koje deluju na procese nukleacije i rasta korozionog piting zamora u četiri vremenska stadijuma, a u cilju izračunavanja radnog veka. Za određivanje verovatnoće je primenjena Montekarlo simulacija zbog njene efikasnosti, /9, 12, 13/.

ISTRAŽIVANJA UZROKA RAZARANJA

U poslednjoj deceniji, saopšteni su rezultati, /1-4, 20/, razaranja turbinskih lopatica usled zamora koji je iniciran na korozionom pitu i kod parnih i kod gasnih turbina, sl. 1-4. Pored zamora, mehanizmi oštećivanja lopatica su mnogostruki: oksidacija i puzanje u uslovima visokog pritiska i korozija u uslovima niskog pritiska. Ipak, u svim slučajevima je istaknuto da je inicijalni piting bio izvor od kojeg je usledio rast prsline usled zamora. Uzimajući u obzir aspekt slučajnosti u odnosu na nivo prsline, njen oblik i brzinu rasta, za rast prsline iz inicijalnog pita usvojen je princip linearno elastične mehanike loma (LEML) na ovom stadijumu, sve do dostizanja izvesne veće brzine rasta prsline: brzine rasta kada dolazi do promene iz sporog u brži rast, koji je označen kao rast duge prsline. Ovaj stadijum je veoma opasan za bilo koju konstrukciju, jer dovodi do veoma brzog statičkog loma.

damage processes – fatigue and corrosion, have their own variability. Both, chemical and mechanical properties could not be gauged accurately in a deterministic sense. Probabilistic aspects combined with mechanical degradation models show reasonable estimates for the damage process and lifespan. Generally, two aspects are considered in fatigue studies: 1. safe life where hypothetically no damage and no crack growth is initiated, and 2. where a growing defect results from load variation.

Presented in this work are all variables that could mutually interact with the damage on the surface of the component and their variability according to data found in literature, and in that sense the stages of fatigue life of the component. The probabilistic model in this paper addresses randomness, affecting nucleation and growth processes in four time stages, for computing the pitting corrosion fatigue life. The Monte Carlo simulation is used in probabilistic estimates, due to its efficiency, /9, 12, 13/.

CASE STUDIES OF FAILURE

In the last decade, research has shown, /1-4, 20/, that turbine blades fail due to fatigue mechanisms with pitting corrosion initiation in both steam and gas turbines, Figs. 1-4. In addition to fatigue, mechanisms of damage in blades are multiple: oxidation and creep in conditions of high pressure, and corrosion at low pressure. Nevertheless, all cases feature pitting initiation as the originating point of fatigue crack growth. Taking into account the random aspect of the level of a crack, with specified shape and rate of growth, the crack grows in this stage, from an initiated pit under the assumption of linear elastic fracture mechanics (LEFM) priciples, until the crack reaches a certain higher level of growth rate: the crack growth rate changes from slow crack growth to fast, called long crack growth. This is a very dangerous state for any structure, since it leads very rapidly to fast static fracture.



Slika 1. Turbina visokog pritiska (TVP), lopatice turbo propelera ALLISON 501-D22, posle 48 000 časova rada bez popravke, /19/ Figure 1. High pressure turbine (HPT), blades of turbo propeller ALLISON 501-D22, after 48 000 hours of service without repair, /19/.



Slika 2. TVP, lopatice turbo propelera ALLISON 501-D22, posle 39 000 časova rada bez popravke, /19/
Figure 2: HPT, turbine blades of turbo propeller ALLISON 501-D22), after 39 000 hours of service without repair, /19/.



Statističke promenljive (Random variables)

Slika 5. Procedura za proračun napona lopatice turbine Figure 5. Procedure for stress calculation of turbine blades.

ODREĐIVANJE OPTEREĆENJA

Da bi se odredio radni vek komponente kada je prisutan kompleksan način oštećivanja, neophodno je naći opterećenja u saglasnosti sa termodinamičkim uslovima, zatim varirati termomehaničko naponsko stanje na osnovu promene termičkih i mehaničkih osobina u funkciji od temperature, sl. 5. U ovom radu, termički ciklus je definisan primenom klasičnih formulacija termodinamičkih zakona u zavisnosti od mašine. Kombinovanjem centrifugalnog efekta i dejstva pritiska, moguće je odrediti uticaj promenljivog mehaničkog napona na osnovu pravila o čvrstoći materijala. Temperaturne promene tokom regularnog rada indukuju termičku deformaciju, a time i promenljive termičke napona u materijalu lopatice.

LOAD DETERMINATION

In order to estimate the component lifetime under complex damage, it is necessary to find the load from thermodynamic conditions and vary the thermomechanical stress state with thermal and mechanical properties as a function of temperature, Fig.5. The thermal cycle is defined in this work by using classical formulations of thermodynamics laws, depending on the engine. By adding the centrifugal effect to the pressure effect, the mechanical varying stresses are found by material strength rules. Temperature variations during normal operation induce thermal deformation, and thus a thermal varying stress in the blade material.

MODEL RADNOG VEKA KOROZIONOG ZAMORA

Ukupan zamorni vek može da se predstavi sumiranjem četiri vremenski zavisna stadijuma, dva prelazna stadijuma i lom prema modelu Ši i Mahadevan, sl. 6, /12/.

- 1. Vreme do inicijacije defekta.
- 2. Rast pita.
- 3. Prelaz od pita do kratke prsline.
- 4. Rast kratke prsline.
- 5. Drugi prelaz koji predstavlja promenu u rastu prsline iz kratke u dugu.
- 6. Rast duge prsline.
- 7. Lom.

MODEL OF PITTING CORROSION FATIGUE LIFE

The total fatigue life may be represented by the summation of four time dependent stages, two transition stages and fracture, in accordance with Shi and Mahadevan model, Fig. 6, /12/.

- 1. Time to defect initiation.
- 2. Pit growth.
- 3. Transition from pit crack to short crack growth.
- 4. Slow crack growth.
- 5. Second transition that represents the variation of crack growth from short to long crack.
- 6. Long crack growth.
- 7. Fracture.



Slika 6. Sedam stadijuma Shi-Mahadevanovog modela tolerancije grešaka Figure 6. Seven stages of Shi-Mahadevan's damage tolerant model.

Inicijacija pita

Vreme određivanja inicijacije oštećenja je problem sa kojim se susreću naučnici i danas, /20, 21/, jer sam uzrok i mehanizam nastanka pita još uvek nisu rasvetljeni, pa samim tim ne može da se odredi ni slučajnost ovog mehanizma. Generalno, inicijacija pita se razmatra kao potpuno slučajan proces, iako je u nekim studijama učinjen pokušaj da se otkrije uzrok koji će da dovede do inicijalnog obrazovanja pita. Prema nekim istraživačima, /1, 20/, inicijacija pita je uslovljena prisustvom uključaka u materijalu. Prema Martinu, /22/, a zahvaljujući posmatranju pod skenirajućim mikroskopom sa sondom (AFM), najverovatnije (70%) se inicijacija pita javlja u nanoplastično deformisanim oblastima.

U ovoj studiji je pretpostavljeno da je vreme do inicijacije pita – vreme sigurnog radnog veka komponente, koje je izračunato primenom Manson–Kofin jednačine (1). Pošto je poznato da lopatice mogu da budu izložene i puzanju i koroziji, onda je puzanje razmatrano primenom Larson– Milerove jednačine, a korozija rešavanjem prema /7/, da bi se odredio radni vek.

Initiation of a pit

Determinating the time to damage initiation is even today a problem for many scientists, /20, 21/, because the reason for, and mechanisms of pit initiation are still not clarified, and as such, the randomness of this mechanism cannot be determined. In general, pit initiation is considered as a totally random process, although in several studies attempts were made to find the cause that would bring about pit development. According to some authors, /1, 20/, pit initiation is caused by the presence of inclusions in the material. Martin concludes, /22/, thanks to atomic force microscopy (AFM), that pit initiation is more likely (70%) on nano-plastically deformed locations.

In this study we assume that the time to pit initiation is the safe lifetime of a component in exploitation, calculated by Manson–Coffin's law, Eq. (1). It is known that blades may be exposed both to creep and corrosion, and so creep is considered by applying the Larson–Miller formulation, and corrosion according to the solution in /7/, in order to determine the service lifetime.

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f}{E} (2N_f)^b + \varepsilon_f (2N_f)^c \tag{1}$$

gde je: $\Delta \varepsilon/2$ -relativna deformacija u toku jednog ciklusa, σ_{f} -visoko ciklična zamorna čvrstoća, *E*-modul elastičnosti, $2N_{f}$ -broj ciklusa, ε_{f} -deformacija zatezanjem, *c* i *b*-konstante.

Da bi se odredila interakcija između oštećenja usled zamora i puzanja, nekoliko pravila o akumulaciji oštećenja može da se koristi, s tim što akumulacija može da se razmatra kao linearna i nelinearna veličina. U okviru ovog rada je primenjeno linearno Majnerovo pravilo o akumulaciji oštećenja, koje pri potpunom oštećenju ima vrednost 1. Potom je određen broj sigurnih ciklusa pri startovanju i zaustavljanju.

Stadijum rasta pita

U ovom stadijumu se javlja rast pita zbog lokalne nehomogenosti, koje su izdvojene u materijalu, i delujućih elektrohemijskih procesa. Parametri koji opisuju Faradejev zakon su ekstremno slučajne veličine, posebno struja pita i geometrijski oblik pita. Turnbul i saradnici, /21/, su istraživali uslove pod kojima prekid pasivnosti (pasivnog filma) interaktivno kreira rast pita. Rajansankar i Nageshijer, /13/, su primenom empirijskih istraživanja i probabilističkih pristupa određivali brzinu rasta pita i njegov oblik. U tom modelu je pretpostavljeno da pit raste pri konstantnoj zapreminskoj brzini, dV/dt, saglasno Faradejevom zakonu, pa je moguće odrediti t_{in} , vreme rasta pita, koje je funkcija većeg broja promenljivih. Znači,

$$\frac{dV}{dt} = \frac{MI_{po}}{nF\rho} \exp\left(-\frac{\Delta H}{RT}\right)$$
(2)

gde je: *M*-molekularna masa materijala, *n*-valentnost, *F*-Faradejeva konstanta, ρ -gustina, ΔH -aktivaciona energija, *T*-apsolutna temperatura *R*-univerzalna gasna konstanta. Za pit je pretpostavljeno da ima eliptičan oblik u beskonačnoj ploči, čija je zapremina *V*:

$$V = \frac{2}{3}\pi ca^2 \tag{3}$$

gde su *a* i *c* polovine dužine velike i male ose elipse.

Određivanje prvog prelaznog stadijuma

Treći stadijum usvojenog modela se odnosi na prelaz rasta pita u rast kratke zamorne prsline, u kom slučaju se u obzir uzima i uticaj faktora koncentracije napona. Usvojeno je da se prelaz javlja kada je:

$$\left(\Delta K\right)_{pit} = \left(\Delta K\right)_{prslina} \tag{4}$$

gde je ΔK amplituda faktora intenziteta napona pita, tj. prsline.

Primenom Faradejevog zakona nalazi se vreme rasta pita, a rešavanjem jednačine prelaza dobija se kritična veličina pita koja učestvuje u prelazu.

Rast kratke prsline

Parisov zakon sa specifičnim parametrima je korišćen u ovom stadijumu da bi se definisala vrednost t_{scg} , koja je funkcija prelaza u rast klasične duge prsline. U određenim studijama, /12, 18/, je istaknuta važnost efekta ovog stadijuma: brzina kratke prsline je veća od brzine duge prsline, dok su naponi manji od onih za rast duge prsline.

$$\frac{da}{dN} = C_{scg} \left(\Delta k\right)^{m_{scg}} \tag{5}$$

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$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f}{E} (2N_f)^b + \varepsilon_f (2N_f)^c \tag{1}$$

where: $\Delta \varepsilon/2$ -relative strain over one full cycle, σ_{f} -high cycle fatigue strength, *E*-Young's modulus, 2*N*_f-number of strain reversals, ε_{f} -close to tensile ductility, *c* and *b*-constants.

In order to consider the interaction between fatigue and creep damage, several rules of damage accumulation may be used, where this accumulation can be considered as linear or nonlinear. The Miner rule is applied here, as the linear damage accumulation rule, that reaches unity for total damage. Subsequently, the safe number of cycles for startup and shut-down, are found.

Pit growth stage

This stage concerns pit growth by local distinct inhomogeneities in the material, and involved electrochemical processes. Parameters in Faraday's law are highly random, especially pit current and pit shape geometry. Turnbull et al, /21/, investigated conditions in which the passivity breakdown (of the passive film) interacts in generating pit growth. Rajansankar and Nageshiyer, /13/, have studied, through empirical investigations and probabilistic approaches, the rate and the shape of pit growth. In this model, the pit is assumed to grow at a constant volumetric rate, dV/dt, according to Faraday's law, and the time of pit growth t_{in} can be determined as a function of a large number of variables. Accordingly,

$$\frac{dV}{dt} = \frac{MI_{po}}{nF\rho} \exp\left(-\frac{\Delta H}{RT}\right)$$
(2)

where: *M*-molecular mass of the materiel, *n*-valence, *F*-Faraday's constant, ρ -density, ΔH -activation energy, *T*-absolute temperature, *R*-universal gas constant. The pit is assumed as a half prolate spheroid in an infinite plate of volume *V*:

$$V = \frac{2}{3}\pi ca^2 \tag{3}$$

where *a* and *c* are the semi-major and -minor ellipse axes.

Determination of the first transition stage

This third stage considers transition from pit growth to short fatigue crack growth, where the effect of the stress concentration factor is taken into account. The transition is assumed to occur when:

$$\left(\Delta K\right)_{pit} = \left(\Delta K\right)_{prslina} \tag{4}$$

where: ΔK -stress intensity amplitude of pit and crack, in respect.

By applying Faraday's law, the pit growth time is calculated, and by solving the transition equation, the critical pit size involved in the transition is also determined.

Short crack growth

The Paris law with specific parameters is used in this stage to define t_{scg} as a function of the crack transition to classical long crack growth. Some studies, /12, 18/, demonstrate the important effect of this stage: short crack rate exceeds long crack rate, while the stresses are below those in long crack growth.

$$\frac{da}{dN} = C_{scg} \left(\Delta k\right)^{m_{scg}} \tag{5}$$

STRUCTURAL INTEGRITY AND LIFE Vol. 8, No 1 (2008), pp. 3–12 gde je: *a*-dužina prsline, *N*-broj promena, C_{scg} -konstanta materijala za kratku prslinu, m_{scg} -eksponent rasta za kratku prslinu.

Određivanje drugog prelaznog stadijuma

U analitičkom pristupu, drugi prelazni stadijum je okarakterisan izjednačavanjem brzina rasta kratke i duge prsline. U ovom radu je drugi prelazni stadijum razmatran na deterministički način, ali sa slučajnim veličinama.

Rast duge prsline

I u ovom stadijumu se koristi klasičan Parisov zakon, uzimajući u obzir konstante materijala za rast duge prsline i faktor intenziteta napona koji važi za prslinu u beskonačnoj ravnoj ploči. Vreme t_{lcg} je izračunato za konačnu kritičnu veličinu duge prsline. Ova vrednost definiše granicu kada treba da se izvrši zamena, npr. 1 mm.

$$\frac{da}{dN} = C_{lcg} \left(\Delta k\right)^{m_{lcg}} \tag{6}$$

gde je: *a*–dužina prsline, *N*–broj promena, C_{lcg} –konstanta materijala za dugu prslinu, m_{lcg} –eksponent rasta za dugu prslinu. *Lom*

Razmatra se prslina koja je neprihvatljive veličine, ali koja je manja od veličine prsline kada nastupa lom. Vreme do loma u uslovima zamora i korozije je tada:

$$T_f = t_{in} + t_{pg} + t_{scg} + t_{lcg} \tag{7}$$

gde je: T_f -vreme provedeno u radu, t_{in} -vreme inicijacije, t_{pg} -vreme rasta pita, t_{scg} -vreme rasta kratke prsline, t_{lcg} -vreme rasta duge prsline.

STATISTIČKI PROMENLJIVE VELIČINE U MODELU

Prikazani model uključuje nekoliko neizvesnih podataka, pri čemu su neki od njih, do danas, još uvek potpuno slučajne veličine. Model je primenjen na specifičan slučaj sa specifičnim promenljivim veličinama čije su vrednosti ili određene ili preuzete iz literature, a dobijene su istraživanjima ili statističkom analizom. Najveću teškoću je predstavljalo određivanje raspodele podataka statistički promenljivih veličina. U modelu su, kroz svih sedam stadijuma, prisutne statističke veličine, npr. raspon napona je jedna od njih, a njihovo određivanje uključuje takođe brojne statističke veličine. U modelu verovatnoće tolerancije oštećenja, najznačajnije statističke veličine su:

- 1. Inicijacija pita vreme do inicijacije, usvojena je lognormalna raspodela sa određenom vrednošću sigurnog veka, koja je izračunata u navedenoj proceduri.
- Rast pita statistički promenljive veličine uključene u ovom stadijumu su konstanta struje, nakupine čestica od kojih zavisi struja pita, geometrija oblika pita, koja takođe prati log–normalnu raspodelu, mikrostruktura materijala, /20–23/.
- Prvi prelazni stadijum u ovom stadijumu su uključene promenljive koje utiču na prelaz iz rasta pita u rast kratke prsline.
- 4. Rast kratke prsline statistički promenljive veličine koje utiču na rast kratke prsline su rasponi napona, faktori koncentracije napona, brzina rasta, usvojena je log-normalna raspodela, iako je u nekim studijama za ovu svrhu korišćena Vejbulova raspodela.

where: *a*-crack length, *N*-number of cycles, C_{scg} -material constant for short crack, m_{scg} -growth exponent for short crack.

Determination of the second transition stage

In the analytical approach, the second transition stage is characterised by equating growth rates of short and long cracks. In this work, the second transition stage is treated in a deterministic way, but with random variables.

Long crack growth

The classic Paris law is also used in this stage, dependent on material constants for long crack growth and on stress intensity factor of a crack in an infinite plate. The time t_{lcg} is calculated for the final critical size of long crack growth. This value defines the time limit for considering repairs, i.e. 1 mm.

$$\frac{da}{dN} = C_{lcg} \left(\Delta k\right)^{m_{lcg}} \tag{6}$$

where: *a*-crack length, *N*-number of cycles, C_{lcg} -material constant for long crack, m_{lcg} -growth exponent for long crack.

Fracture

A crack of unacceptable size is considered, but one shorter than the crack size when real fracture occurs. The time to fracture in conditions of fatigue corrosion is then:

$$T_f = t_{in} + t_{pg} + t_{scg} + t_{lcg} \tag{7}$$

where: T_{f} -service time, t_{in} -initiation time, t_{pg} -pit growth time, t_{scg} -growth time of short crack, and t_{lcg} -long crack growth time.

RANDOM VARIABLES IN THE MODEL

The model above includes several uncertain data and yet, some data are still totally random variables. The model is applied for a specific case, whose specific variables are either determined or taken from literature, but obtained from empirical research or by statistical data analysis. Most difficulties had emerged in defining the statistical distribution information about the random variables. Statistical data are inevitable in the model in all seven stages, e.g. the stress range, and their evaluation implicates also numerous random variables. The most significant random variables in the damage tolerance model are:

- 1. Pit initiation time to initiation, and is taken as the lognormal distribution with predetermined value of the safe lifespan, calculated in the above procedure.
- 2. Pit growth random variables involved in this stage are: pit current constant, number of clustered particles upon which the pit current depends, pit shape geometry that seems to obey the log–normal distribution, and material microstructures, /20–23/.
- 3. First transition stage the transition stage includes variables that affect transition from pit growth to short crack growth.
- 4. Short crack growth random variables that affect short crack growth are: stress ranges, stress concentration factors, growth rate, log–normal distribution is assumed, despite that the Weibull distribution is used in some studies.

- 5. Drugi prelazni stadijum u ovom stadijumu su uključene promenljive koje utiču na prelaz iz rasta kratke prsline u rast duge prsline, s tim što se ovaj stadijum naslanja na prethodni i naredni stadijum.
- 6. Rast duge prsline ovaj stadijum zavisi od raspona napona, faktora koncentracije napona i brzine rasta.
- 7. Lom ovaj stadijum zavisi od zaostalih napona i raspona napona.

METODE ANALIZE VEROVATNOĆE

Granično stanje, odnosno ovde data analiza, je izvedena radi izračunavanja kumulativne funkcije raspodele radnog veka u uslovima korozionog zamora. Za vremena, delove radnog veka, koja karakterišu četiri glavna stadijuma, je usvojena log–normalna raspodela sa srednjom vrednošću izračunatom determinističkim proračunom. Funkcija graničnog stanja je određena kao:

$$G(X_i) = 0 \tag{7}$$

$$t_{in} + t_{pg} + t_{scg} + t_{lcg} \le t_f \tag{8}$$

$$G(X_i) = t_{in} + t_{pg} + t_{scg} + t_{lcg} - t_f$$
(9)

gde su: X_t -statistički promenljive veličine t_{in} , t_{pg} , t_{scg} , t_{lcg} i t_f , pri čemu je t_f -vreme provedeno u radu.

Za ovaj model je usvojeno da je proračun pouzdanosti na osnovu momenata prvog i drugog reda u skladu sa pretpostavljenom ravnomernom i normalnom raspodelom, iako to nije opšti slučaj. Veoma često problem verovatnoće nije ni linearan, ni kontinualan. Zbog tačnosti, jednostavnosti i valjanosti, korišćena je Montekarlo simulacija, /12, 13/.

Montekarlo simulacija je korišćena da bi se odredila verovatnoća loma većeg broja uzoraka primenom jedn. (10), dok je zahtevana veličina uzorka za pretpostavljenu grešku (ε , %) izračunata primenom jedn. (11),

$$P_{Def} = \frac{broj \ G(X_i) \le 0}{N_{pokusaja}}$$
(10)

$$\varepsilon\% = \sqrt{\frac{1 - P_f^T}{NP_f^T}} \times 200\%$$
(11)

gde su: P_f^T -stvarna verovatnoća pojave loma, *N*-broj simuliranih uzoraka.

PRIMENA

Prikazan primer pokazuje istraživanja verovatnoće loma lopatice Alison turbo reaktora, izrađene od superlegure na bazi nikla, sl. 3. Na lopatici je vidljivo razaranje koje se desilo pre prvog planiranog remonta turbo reaktora. Ocenjeni delovi radnog veka ("životi") nađeni za ovaj slučaj, izračunati u /5, 11/, su dati u tabeli 1. Dalje, učinjen je pokušaj da se izvrši simulacija verovatnoće do loma ispitivanih lopatica za nekoliko delova radnog veka. Funkcija graničnog stanja $G(X_i)$ je određena sa vremenom kao statistički promenljivom veličinom. Dve vrste raspodele su primenjene da bi se odredila verovatnoća do loma – normalna raspodela, sl. 7a i log–normalna raspodela, sl. 7b. Veoma velika verovatnoća loma je nađena i za vrlo kratka vremena rada, što može da ukaže na konstrukcijske nedostatke ovog tipa lopatica. Efikasnost Montekarlo simulacije može da se vidi kroz rezultate

- 5. Second transition stage this stage includes variables affecting transition from short crack growth to long crack growth, whereupon this step is dependent on the former and the following stages.
- 6. Long crack growth this stage depends on stress ranges, stress concentration factors, and on growth rate.
- Fracture this stage depends on residual stresses and stress ranges.

METHODS FOR PROBABILISTIC ANALYSIS

The limit state, i.e. the analysis given here, is derived in order to compute the cumulative function of the lifetime distribution in conditions of corrosion fatigue. Random variables in this model are time fractions of four main stages and their distribution is assumed as log-normal with the mean value calculated by deterministic calculations. The limit state function is defined by:

$$G(X_i) = 0 \tag{7}$$

$$t_{in} + t_{pg} + t_{scg} + t_{lcg} \le t_f \tag{8}$$

$$G(X_i) = t_{in} + t_{pg} + t_{scg} + t_{lcg} - t_f$$
(9)

where: X_i -random variables t_{in} , t_{pg} , t_{scg} , t_{lcg} and t_f . The t_f is service time.

This model assumes that reliability calculations, based on moments of the first and second order, and deals with uniform and normal distributions, despite that this is not the general case. Very often, the probabilistic problem is nonlinear and not continuous. For sake of accuracy, simplicity and validation purposes, the Monte Carlo simulation is used, /12, 13/.

The Monte Carlo simulation is used to estimate the probability of rupture for a large number of samples, according to Eq. (10), while the required sample amount for an assumed error (ε %) is calculated from Eq. (11),

$$P_{Def} = \frac{number \ G(X_i) \le 0}{N_{tries}}$$
(10)

$$\varepsilon\% = \sqrt{\frac{1 - P_f^T}{NP_f^T}} \times 200\% \tag{11}$$

where: P_f^T -true probability of failure, *N*-number of simulated samples.

APPLICATION

The presented example displays an investigation of the probability of failure of Allison turbo reactor blades, made of nickel-based superalloy, Fig. 3. Visible damage on the blade occurred prior to the first scheduled servicing of the turbo reactor. Evaluated lifetime fractions ("lives") in this application, calculated in /5, 11/, are given in Table 1. Hence, an attempt is made to simulate the probability of failure of the tested blades for a set of lifetime fractions. The limit state function $G(X_i)$ is defined with time as the random variable. Two distribution types are applied for estimating failure probability – normal distribution, Fig. 7a, and log–normal distribution, Fig. 7b. Very high probabilities of failure, even for short service times, indicate the short-comings in the design of these blade types. The efficiency of the Monte Carlo simulation is evident from the results

INTEGRITET I VEK KONSTRUKCIJA Vol. 8, br. 1 (2008), str. 3–12 prikazane na sl. 8 i u tabeli 2, koji su dobijeni povećanjem broja pokušaja uz stabilizaciju na 0,01% na milion pokušaja.

Tabela 1.	Veličina	oštećenia tokom	četiri stadijuma	radnog veka
		J	· · · · · · · · · · · · · · · · · · ·	

Stadijum	Inicijacija	Rast pita	Rast kratke	Rast duge
	pita		prsline	prsline
Broj ciklusa	10666	3848	35479	35479
Dužina ošte-		1.00E-03	1	1000
ćenja (µm)		-,	-	



shown in Fig. 8 and in Table 2, that have been acquired by increasing the number of tries and stabilized to 0.01% for a million tries.

Stage	Pit	Pit growth	Short crack	Long crack
	initiation		growth	growth
Cycle Number	10666	3848	35479	35479
Defect length (um)		1.00E-03	1	1000
(mµ) 8.0 6.0 6.0 7.0 8.0 6.0 8.0 8.0 8.0 8.0 8.0 8.0 8.0 8.0 8.0 8	96 965	97 976 90		
≥ _{0.1} ∟ 9.55	9.6 9.65 log-norm	9.7 9.75 9.8 al vreme rada	9.85 9.9 a (service time	9.95 10 9)

Table 1. Damage extent in the four stages of service life.

Slika 7. Verovatnoća do loma za normalnu raspodelu (a) i log-normalnu raspodelu (b), u funkciji vremena rada. Figure 7. Probability of failure for normal distribution (a) and log-normal distribution (b) of service time.



Slika 8. Montekarlo simulacija vremena do loma za normalnu raspodelu i za dva slučaja log-normalne raspodele Figure 8. Monte Carlo simulation of time to fracture for normal distribution and for two cases of log-normal distribution.

μ za log-normalnu raspodelu	9,6	9,7	9,8	9,9	10
Greška u 10e3 pokušaja, %	1,974	1,755	1,5748	1,3416	1,0198
Greška u 10e6 pokušaja, %	0,0019	0,0017	0,0015	0,0011	0,0004

Table 2. Model error.

μ for log–normal distribution	9.6	9.7	9.8	9.9	10
% Error for 10e3 tries	1.974	1.755	1.5748	1.3416	1.0198
% Error for 10e6 tries	0.0019	0.0017	0.0015	0.0011	0.0004

ZAKLJUČCI

U ovom radu su određene najvažnije promenljive i parametri koji utiču na razvoj oštećenja usled zamora kod lopatica. Primenjen je verovatnosni pristup za četiri osnovna stadijuma razvoja oštećenja u vremenu preko funkcije graničnog stanja. Istaknuto je da su prva dva stadijuma u kojima dolazi do inicijacije oštećenja veoma značajna i da oni imaju veliki uticaj na pouzdanost komponente. Stadijum inicijacije pita do danas još uvek nije razjašnjen i potpuno je slučajan. Rast pita, takođe, još uvek do danas nije u potpunosti poznat. U trećem stepenu je uzeto u obzir najduže vreme dela radnog veka na kojem treba da se zasniva i održa-

CONCLUSIONS

In this study, the most important variables and parameters that affect the development of fatigue damage in blades are evaluated. A probabilistic approach for four time dependent main stages of damage development is applied by assuming a limit state function. It is noted that initiation of damage occurs in the first two stages that are very significant and with considerable effect on component reliability. The pit initiation stage is yet not clarified and is totally random. Pit growth is also not yet fully understood. The third step assumes the longest lifespan fraction for scheduling maintenance and optimal inspection and repair times, in general,

vanje kroz određivanje optimalnog vremena kontrole i opravke, generalno, odgovarajućim metodama. Simulacija Montekarlo je urađena primenom Matlab 7.

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by applying proper methods. The Monte Carlo simulation is performed with the Matlab 7 probabilistic framework.

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