O viskoplastičnim polikristalima prividno inkrementalnog tipa (nastavak)

Milan V. Mićunović

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2. Deo

zasnovan na radu u Phi. Mag. 2005. (ref. [7])

Part A: THEORY

A1. Geometric issues towards evolution equations

$$RVE - plastic\ distortion\ (free\ meso - rotation)$$

$$micro - plastic\ distortion\ (constrained\ micro - rotation)$$

$$residual \quad micro - elastic\ stretch$$

$$\mathbf{\Pi}_{\Lambda P} := \mathbf{1} + \sum_{\alpha} \gamma_{\alpha\Lambda} \mathbf{A}_{\alpha\Lambda} \qquad micro - plastic\ \ distortion\ (only\ by\ slips)$$

$$\mathbf{C}_P = \left\langle \mathbf{C}(\mathbf{\Pi}_\Lambda) \right\rangle = \left\langle \mathbf{\Pi}_\Lambda^T \mathbf{\Pi}_\Lambda \right\rangle \equiv \frac{1}{\Delta V} \sum_{\alpha} \mathbf{\Pi}_\Lambda^T \mathbf{\Pi}_\Lambda \Delta V_\Lambda$$

macroplastic deformation tensor by means of microplastic deformation tensors for individual grains

$$\mathbf{F}_P = \mathbf{C}_P^{1/2}$$

$$D\mathbf{\Pi}_{\Lambda P} = \sum_{\alpha} \mathbf{A}_{\alpha\Lambda} D\gamma_{\alpha\Lambda} + \gamma_{\alpha\Lambda} D\mathbf{A}_{\alpha\Lambda}$$

$$D\mathbf{R}_\Lambda = \boldsymbol{\Omega}_\Lambda \mathbf{R}_\Lambda \qquad \textit{constrained micro-spin}$$

$$\mathbf{A}_{\alpha\Lambda}(t) = \mathbf{R}_\Lambda(t)\mathbf{A}_{\alpha\Lambda}(t_0)\mathbf{R}_\Lambda^T(t)$$

⇓

$$D\mathbf{A}_{\alpha\Lambda} = \boldsymbol{\Omega}_\Lambda \mathbf{A}_{\alpha\Lambda} + \mathbf{A}_{\alpha\Lambda}\boldsymbol{\Omega}_\Lambda^T$$

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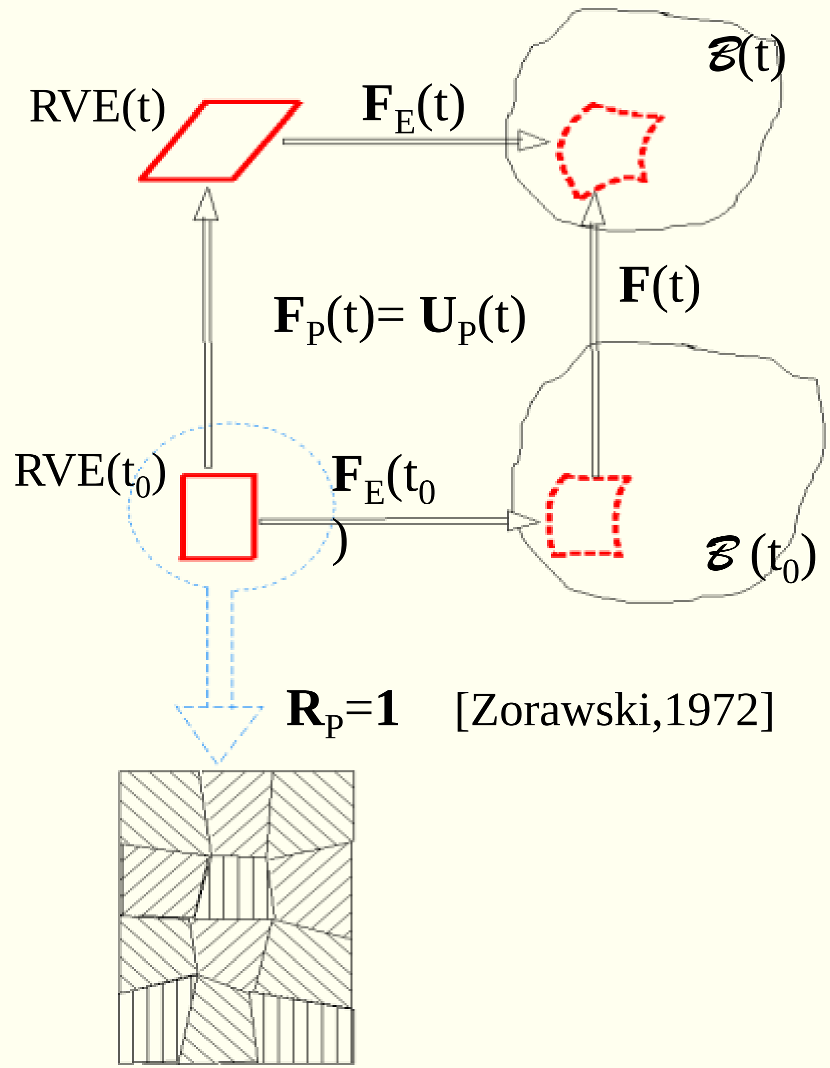
⇓

$$D\mathbf{C}_P = \langle D\mathbf{C}(\boldsymbol{\Pi}_\Lambda)\rangle$$

⇓



Micro ⇒ macro evolution equations

Neale &Toth & Jonas (J2-model) - reference [7]

$$d\gamma_{\alpha\Lambda} = d\gamma_0 \left(\tau_{\alpha\Lambda} / \tau_0\right) \left|\tau_{\alpha\Lambda} / \tau_0\right|^{m-1}$$

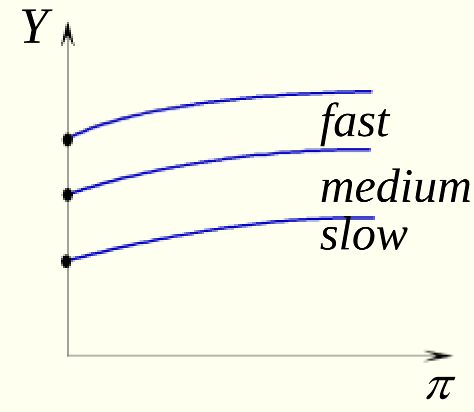
Rate-independent model
(insensitive on initial yield stress Y)

$$\tau_0 = mY_0 \quad (\text{initial shear yield stress} \quad m - \text{orientation factor})$$

Micunovic & Albertini & Montagnani model – reference [6]

$$d\gamma_{\alpha\Lambda} = \pi^{\lambda} \exp(-M)\, sign(\tau_{\alpha\Lambda})\, (\tau_{\alpha\Lambda} - Y)^{\lambda}\, d\sigma_{eq}$$

$$Y = \tau_0 \left[1 + b \ln(D\sigma_{eq})\right]^{\lambda}$$



A2. Evolution equation of quasi-rate independent materials

Tensor representation

$$\mathbf{D}_P = D\sigma_{eq} J(0)\pi^{\beta}\left[c_1 \mathbf{S}_d Y_0^{-1} + c_1 \left(\mathbf{S}_d^2\right)_d Y_0^{-2}\right]$$

Accumulated plastic strain

$$\pi(t) = \int_0^t D\pi(t')\,dt' = \int_0^t J(t-t')\,D\sigma_{eq}(t')\,dt'$$

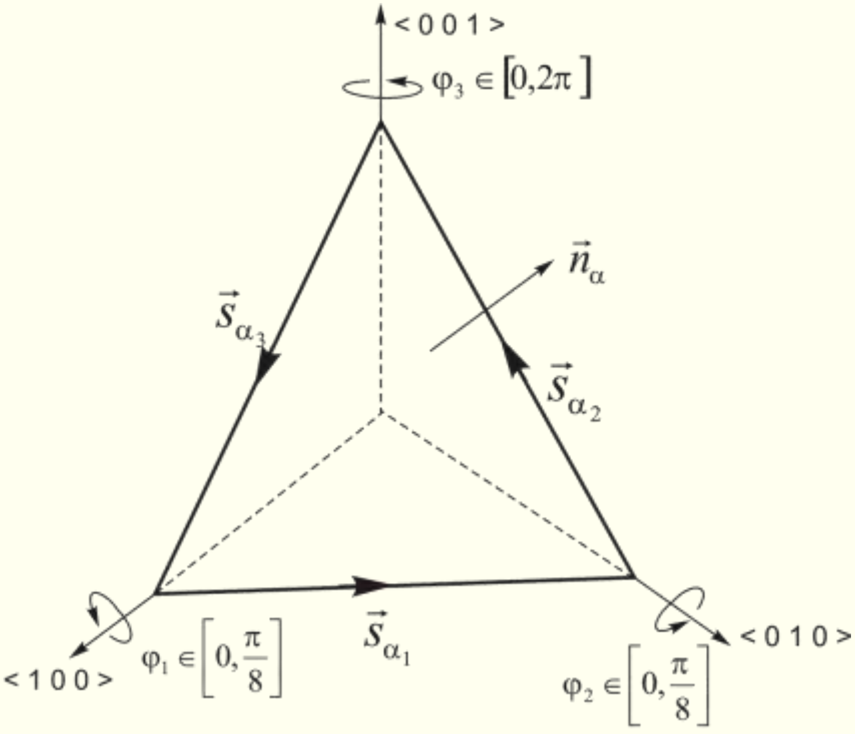
Triggering kernel (slide 5)

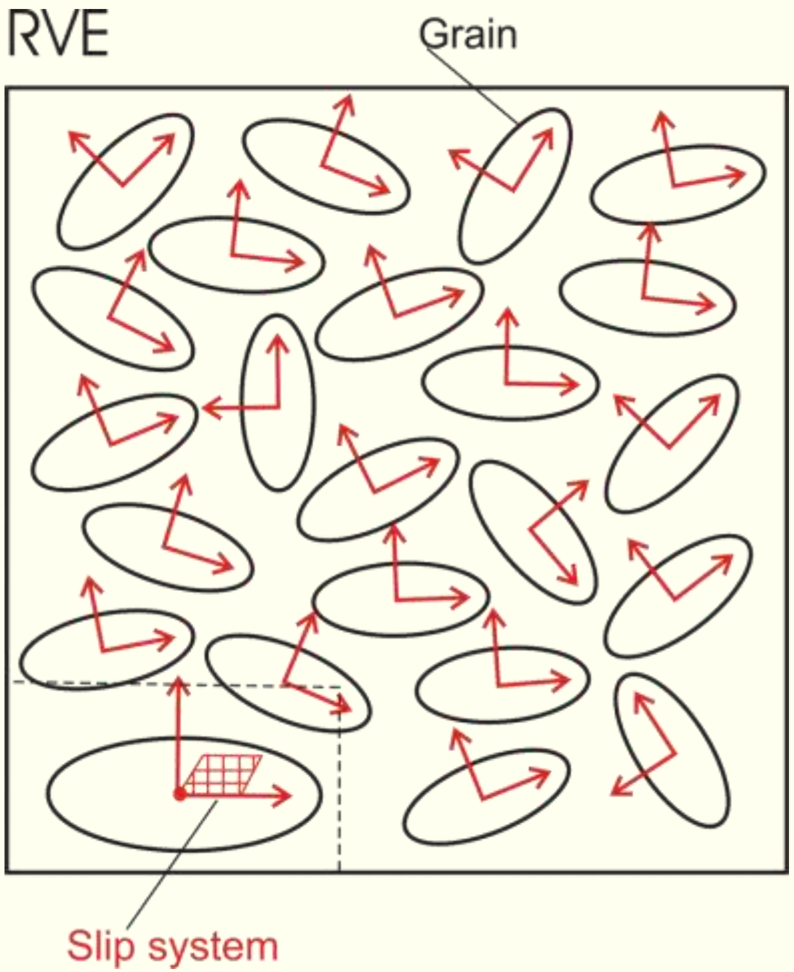
$$J(t-t') = \begin{cases} 0, & t' \geq t_* \\ \exp(-M), & t' < t_* \end{cases}$$

Important note – for loading function $\Omega = \frac{1}{2} h_{\alpha\beta}\tau_\alpha\tau_\beta + A\tau_\alpha\tau_{\alpha 1} + B\tau_\alpha\tau_{\alpha 2}$

normality $\mathbf{D}_P = \Lambda\,\partial_{\mathbf{S}}\Omega$ does not hold

Part B: Numerical experiments on slightly disordered random grain distributed RVE-s





Numerical procedure of integration of field equations

Choice of constitutive model
NTJ-model or MAM-model

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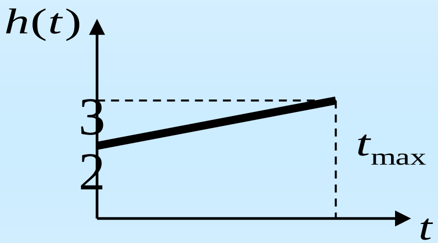
$N_g = 5^3$ *grains* $\quad N_s = 12$ *slip systems*: $\Lambda \in \{1, N_g\}, \quad \alpha \in \{1, N_s\}$

$\mathbf{R}^0_\Lambda$ (*random grain rotations*) $\qquad \mathbf{A}^0_{\alpha\Lambda} = \vec{s}_{\alpha\Lambda} \otimes \vec{n}_{\alpha\Lambda}$

⇓

Stress history



$T_{(0)11} - tension, \ldots, T_{(0)12} - shear$

$$\mathbf{T}_0 = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ldots, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

$\mathbf{T}(t) = \mathbf{T}_0 \, h(t) \, Y_0 \qquad t_{\max} \in \left\{10^2, 10^{-1.5}, 10^{-5}\right\} [s]$

⇓

$\mathbf{T}_{local} = \mathbf{R}_\Lambda \mathbf{T} \mathbf{R}^T_\Lambda \qquad \tau_{\alpha\Lambda} = \mathbf{T}_{local} : \mathbf{A}_{\alpha\Lambda}$

⇓

$$\mathbf{\Pi}_{\Lambda P}(t+\Delta t)=\mathbf{\Pi}_{\Lambda P}(t)+\sum_{\alpha} d\gamma_{\alpha\Lambda}\mathbf{A}_{\alpha\Lambda}+\sum_{\alpha}\gamma_{\alpha\Lambda}d\mathbf{A}_{\alpha\Lambda}$$

$$\mathbf{C}(\mathbf{\Pi}_{\Lambda})\approx\mathbf{\Pi}_{\Lambda}^{T}\mathbf{\Pi}_{\Lambda} \quad \Rightarrow \quad \mathbf{C}_P=\langle\mathbf{C}(\mathbf{\Pi}_{\Lambda})\rangle \quad \textit{volume aveaging}$$

Here $\mathbf{C}_P^{1/2}=\mathbf{U}_P=\mathbf{F}_P \quad \Leftarrow \quad \mathbf{R}_P=\mathbf{1} \quad (M.Zorawski)$

Re*laxed Taylor's assumption* $\mathbf{\Pi}_{\Lambda E}=\mathbf{U}_P\mathbf{\Pi}_{\Lambda P}^{-1}$

$$\mathbf{U}_{\Lambda E}=\left(\mathbf{\Pi}_{\Lambda E}^{T}\mathbf{\Pi}_{\Lambda E}\right)^{1/2}$$

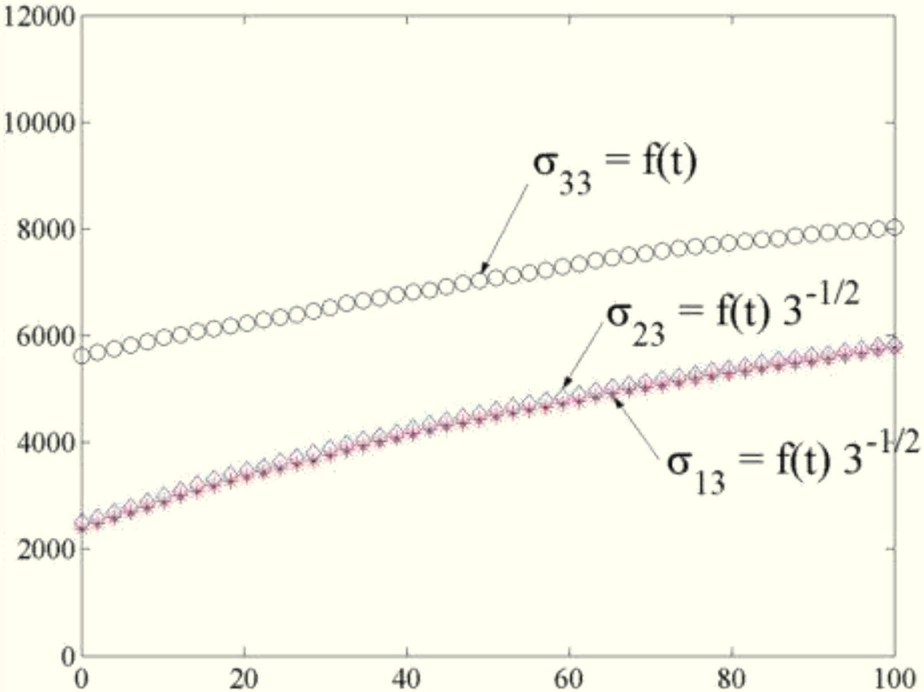
Micro rotation $\mathbf{R}_{\Lambda}=\mathbf{\Pi}_{\Lambda E}\mathbf{U}_{\Lambda E}^{-1}$

Micro spin $\mathbf{\Omega}_{\Lambda}=\left(\mathbf{R}_{\Lambda}(t+\Delta t)-\mathbf{R}_{\Lambda}(t)\right)/\Delta t$

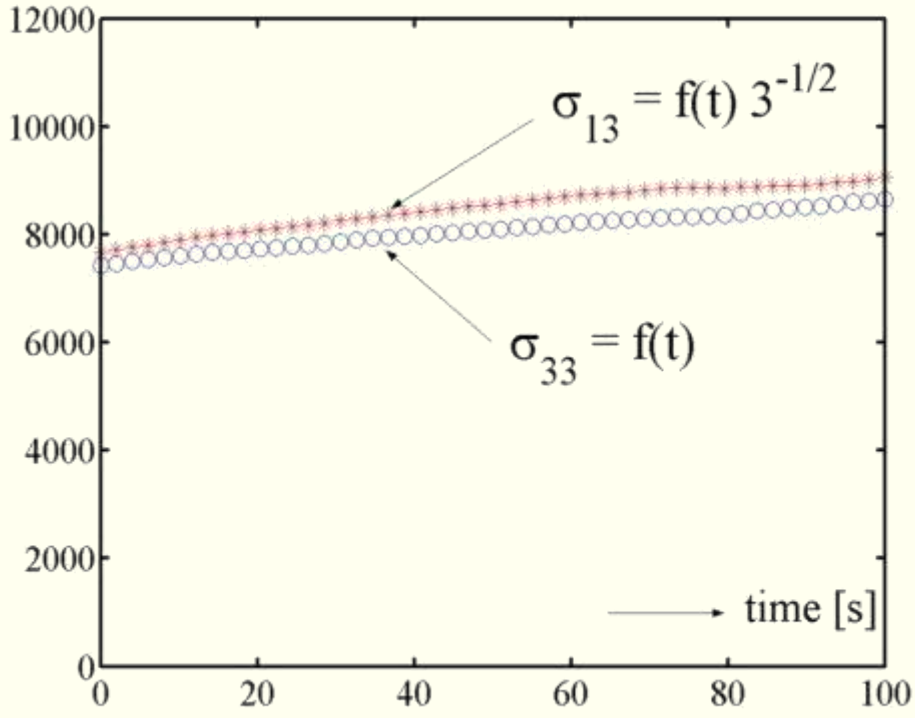
$$d\mathbf{A}_{\alpha\Lambda}=\mathbf{\Omega}_{\Lambda}\mathbf{A}_{\alpha\Lambda}+\mathbf{A}_{\alpha\Lambda}\mathbf{\Omega}_{\Lambda}^{T}$$

Number of active slip systems
Low speed loading

NTJ (J2-model)

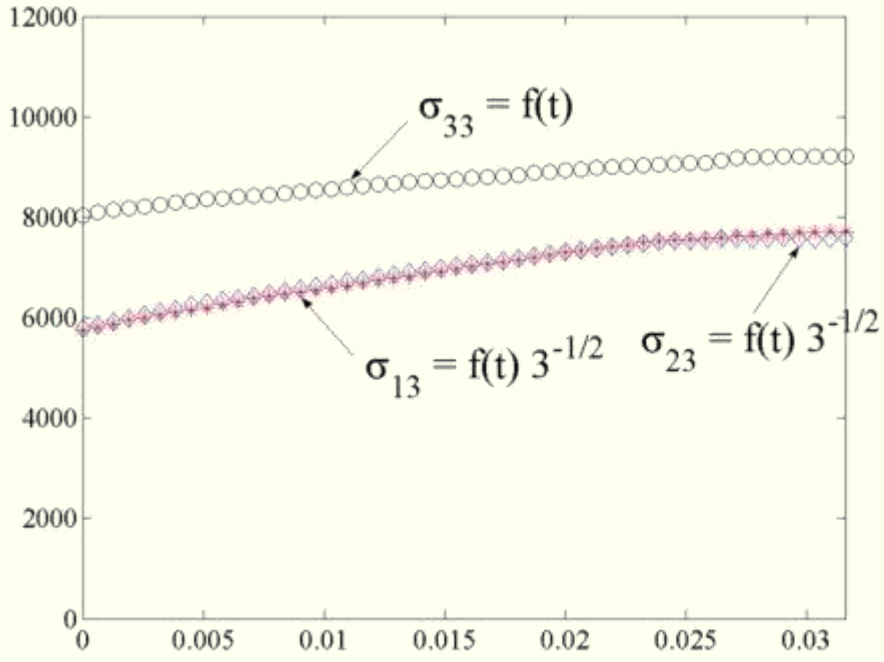


Quasi-rate independent

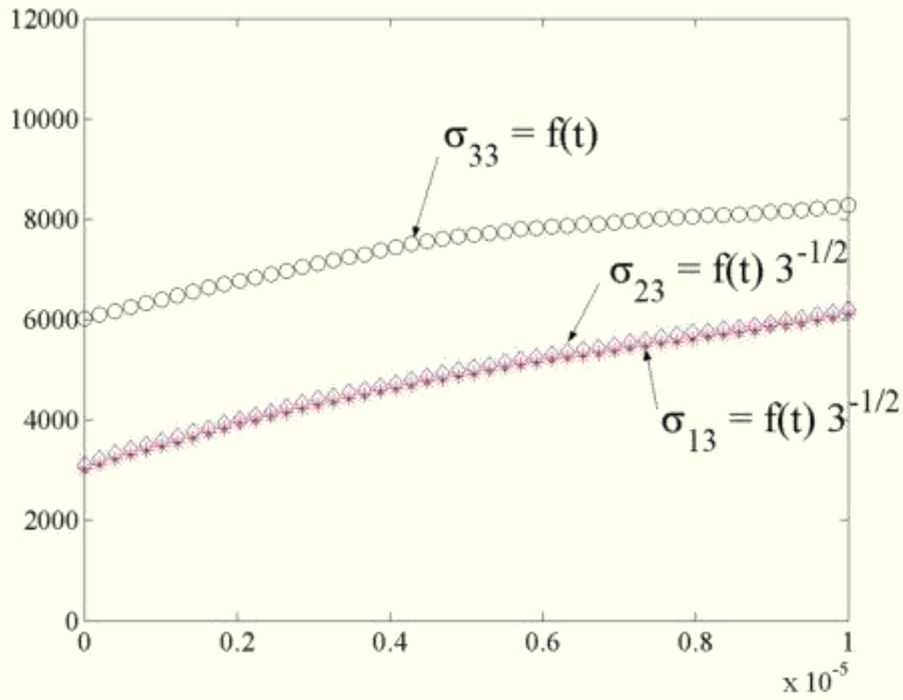


Number of active slip systems
Quasi-rate independent model MAM

Medium speed

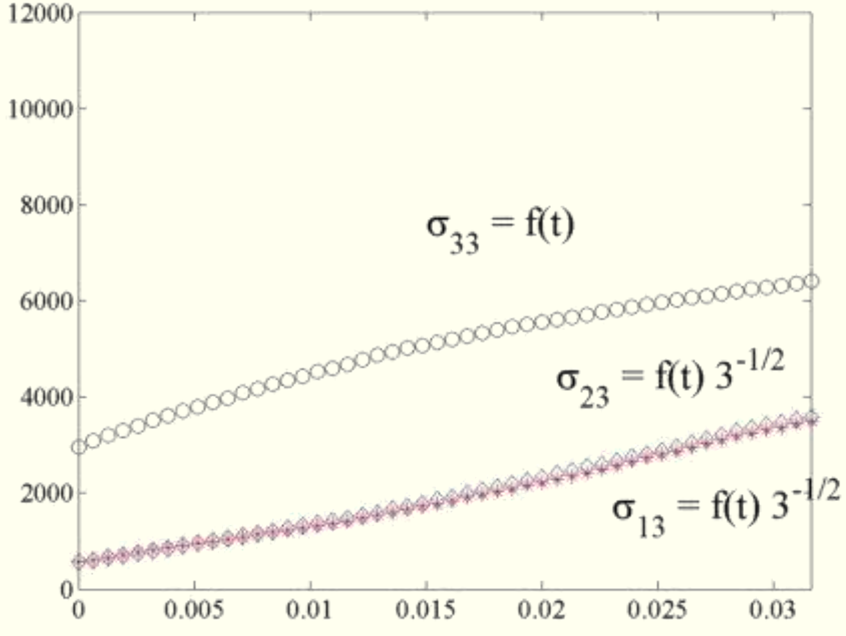


High speed

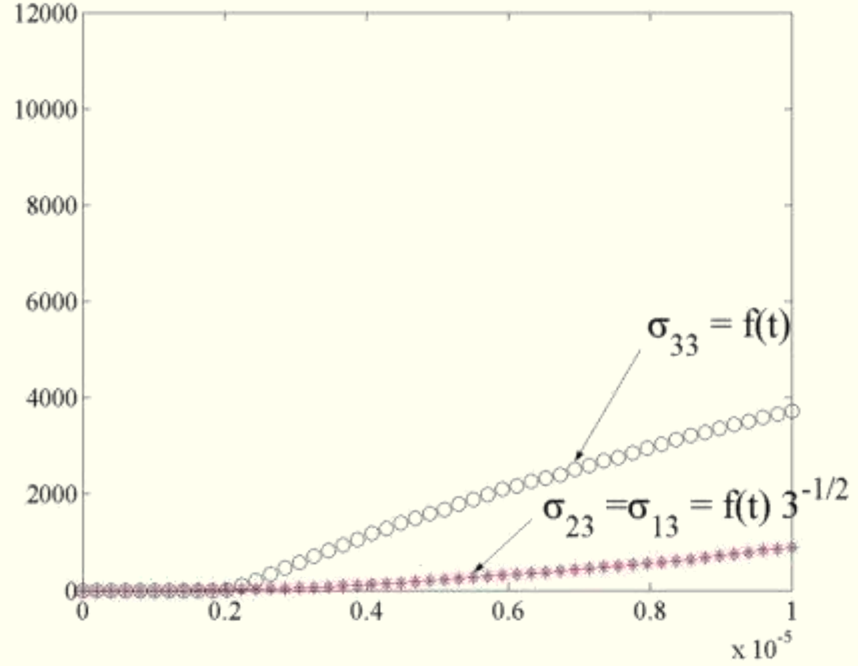


Number of active slip systems
Neale & Toth & Jonas (J2-model) + artificial (Dg,Y)-sensitivity

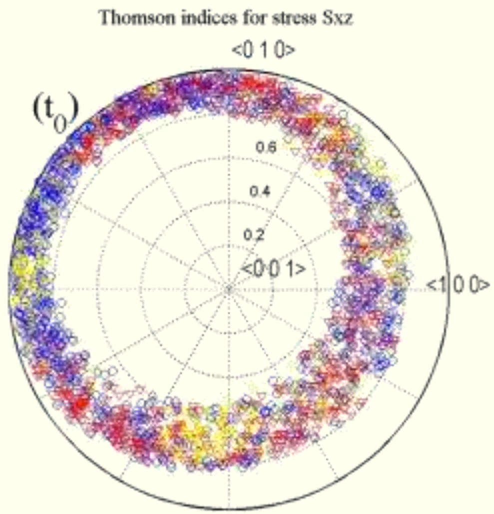
Medium speed

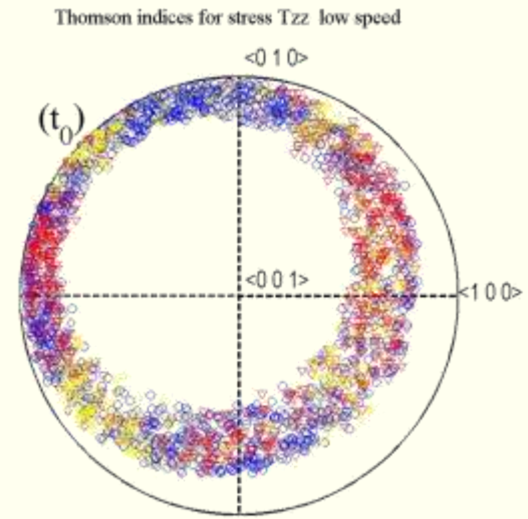


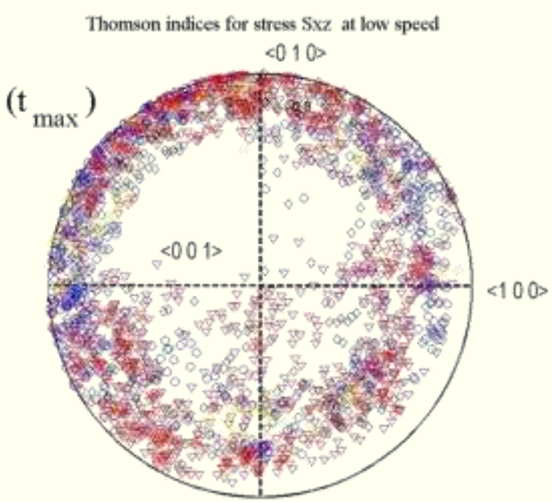
High speed

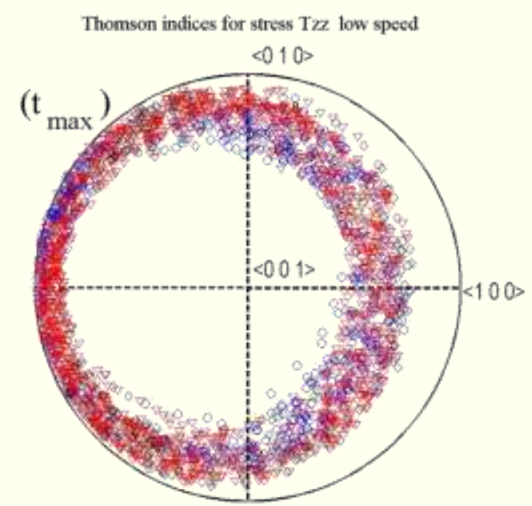


Active slip systems distribution at low speed
Quasi-rate independent model

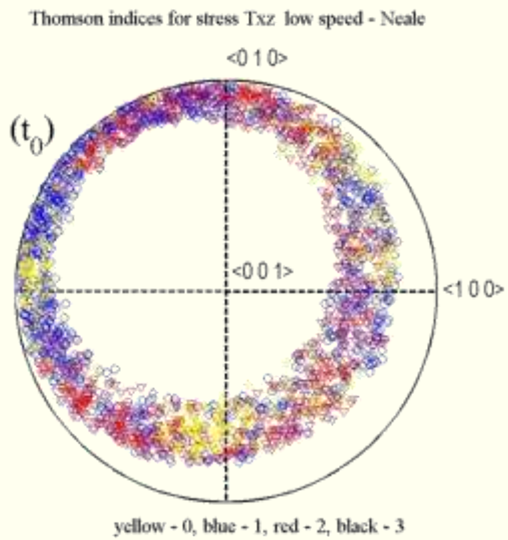


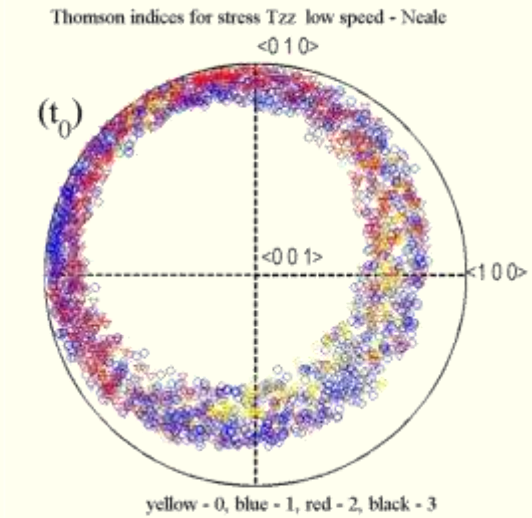


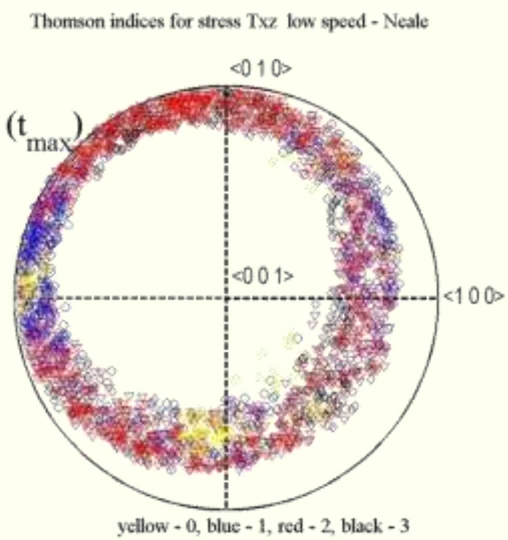


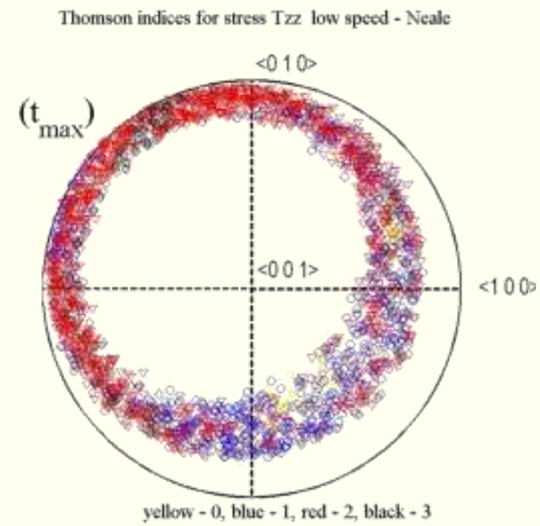


Active slip systems distribution at low speed
Neale & Toth & Jonas (J2-model)



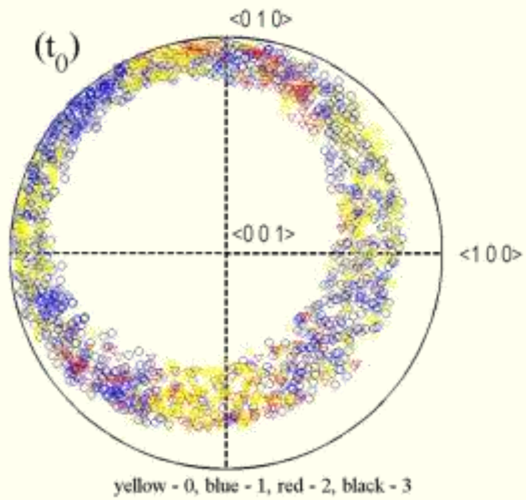




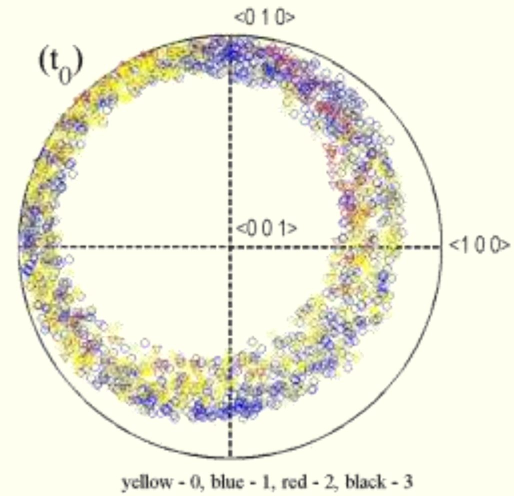


Active slip systems distribution at medium speed
Quasi-rate independent model

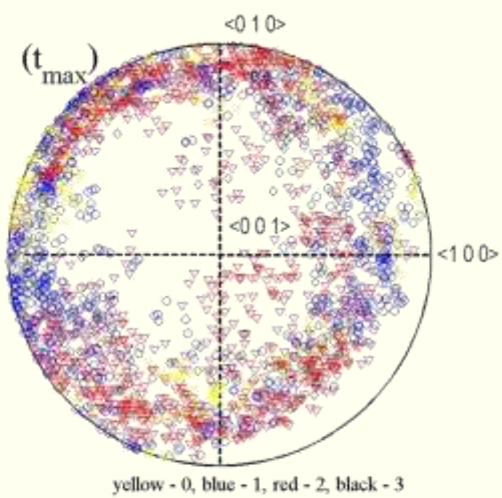
Thomson indices for stress Txz medium speed - MAM



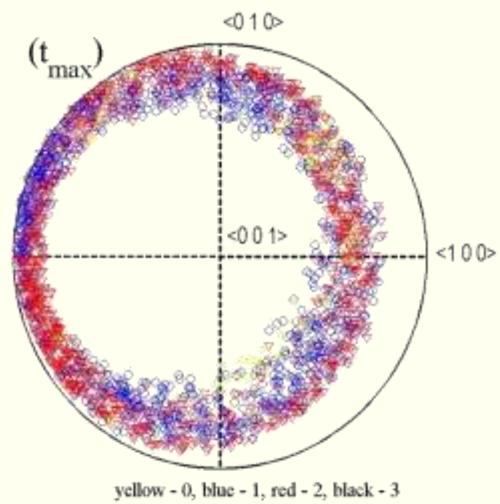
Thomson indices for stress Tzz medium speed - MAM



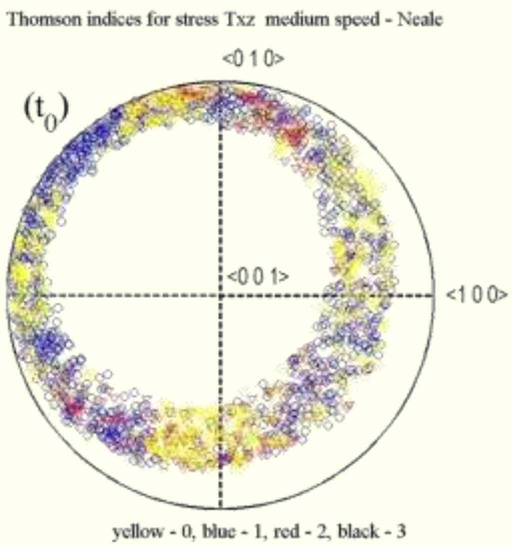
Thomson indices for stress Txz medium speed - MAM

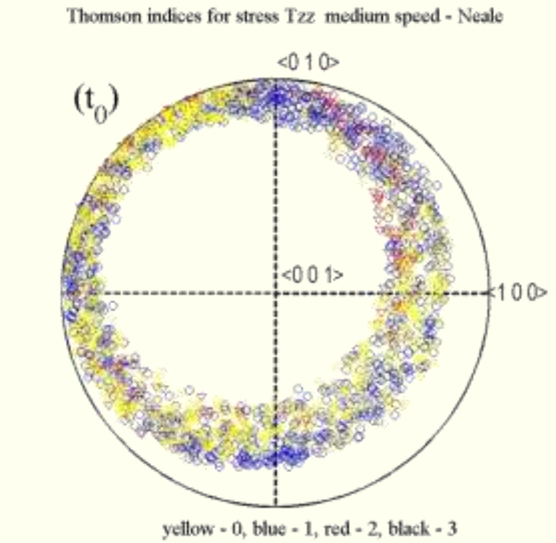


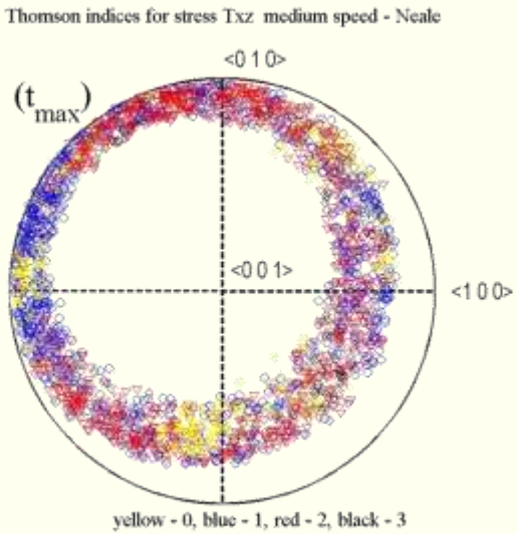
Thomson indices for stress Tzz medium speed - MAM

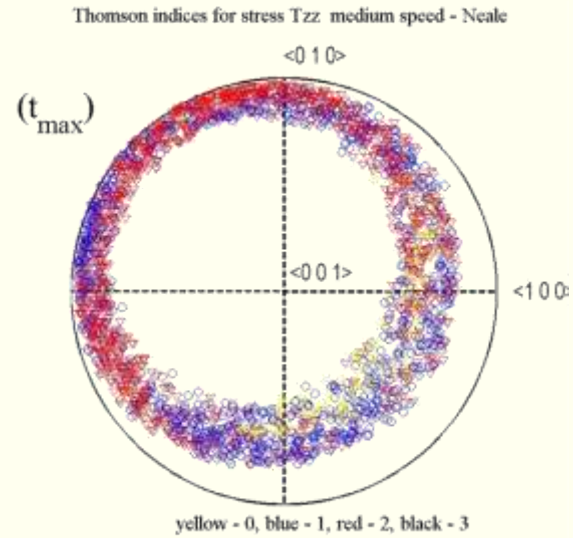


Active slip systems distribution at medium speed
Neale & Toth & Jonas (J2-model)

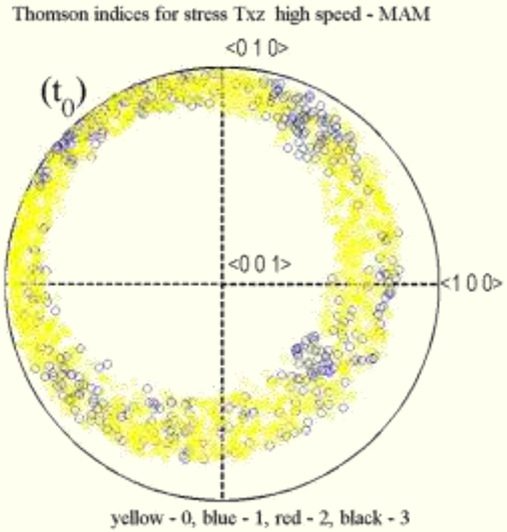


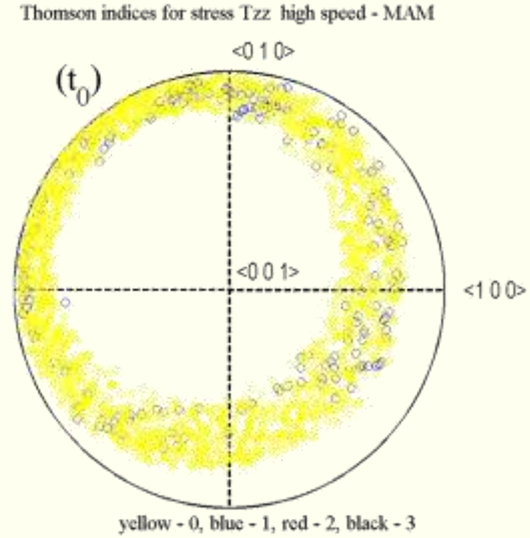


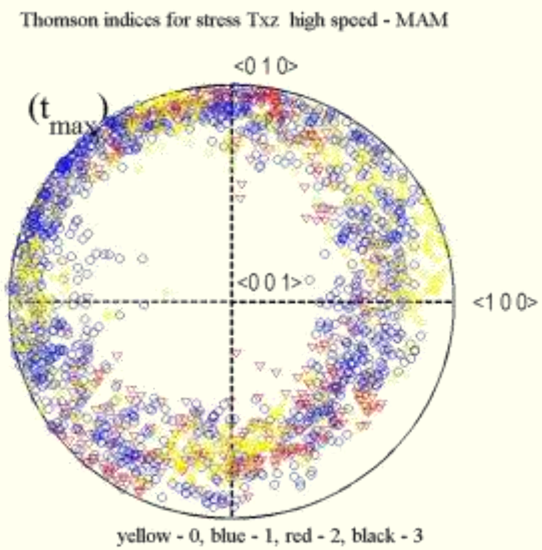


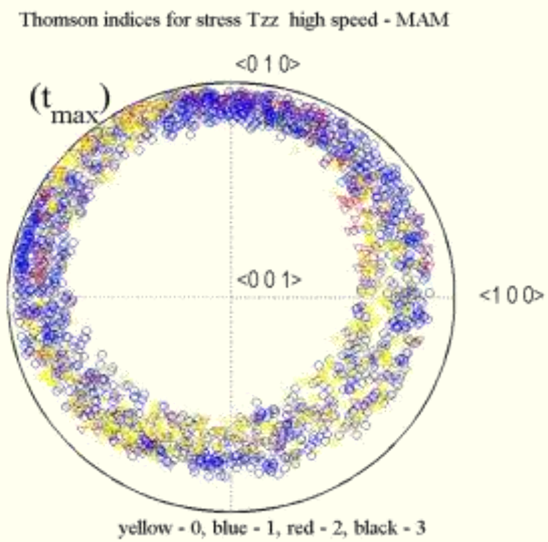


Active slip systems distribution at high speed
Quasi-rate independent model

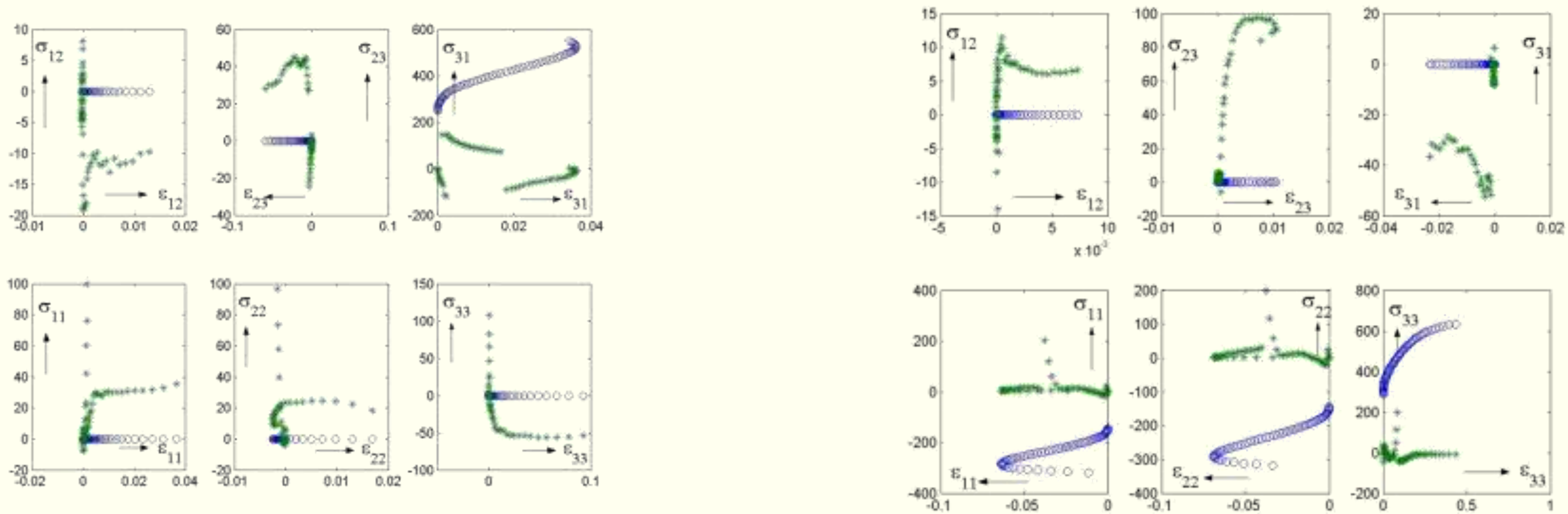




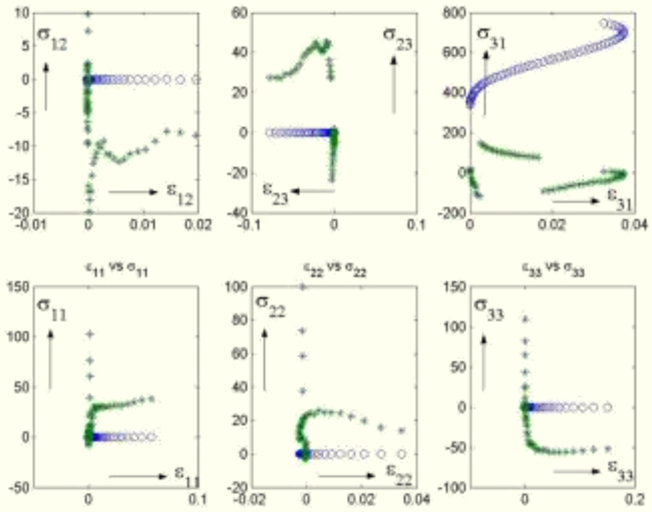


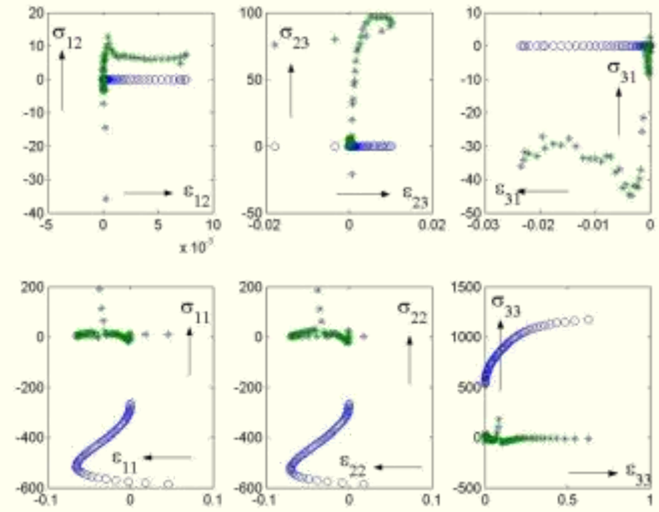


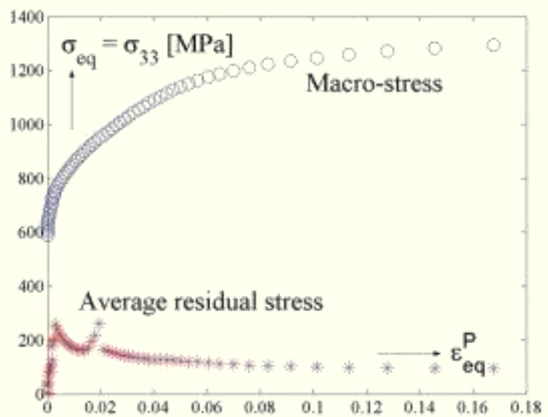
Macroscopic and average residual stresses for xz-shear and zz-unitension at low speed (~0.001/s)
Quasi-rate independent model

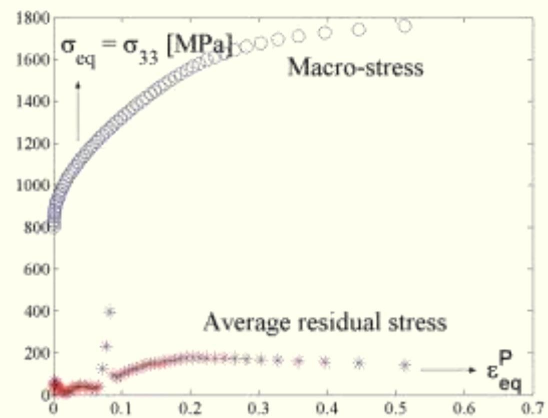


Macroscopic and average residual stresses for xz-shear and zz-unitension at medium speed (~1/s)
Quasi-rate independent model









Concluding remarks

- Evolution equation formed by **tensor representation** having incremental form is postulated to model inelastic metals. The rate dependence takes place by means of stress rate dependent value of the initial yield stress. This approach for auste-nitic stainless steels has permitted exceptionally good agreement with dynamic experiments performed at JRC-Ispra, Italy at strain rates in the interval [0.001, 1000] 1/s.
- This theory is applied to slightly disordered fcc-polycrystals. For some characteristic given stress histories (leading to low, medium and high strain rates) number of active slip systems and average Taylor's m-number for RVE with 1000 grains are found and compared with so-called J2-approach.
- Possible practical applications could be:
  - Calculation of grain interactions where grains have crystals of diverse orientations (Hungarian 3 X 3 X 3 cube) by FEM
  - Calculation of residual stresses in heat affected zone and welded materials composed of grains of diverse size

References

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